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ABSTRACT

Unified Technical Concepts (UTC) is a modular system for teaching applied physics in two-year postsecondary technician programs. This UTC classroom textbook, consisting of 10 chapters, deals with mathematical concepts as they apply to the study of physics. Addressed in the individual chapters of the text are the following topics: angles and triangles; vectors and scalars; reading and drawing graphs; logarithms and exponents, precision, accuracy, and measurements; laboratory notebook preparation; and sinusoidal waves. Each chapter contains instructional objectives, instructional text, a summary, a list of references, and review questions. (MN)

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MATH FOR TECHNICIANS

CENTER FOR OCCUPATIONAL

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601 LAKE AIR DRIVE

WACO , TEXAS 76710

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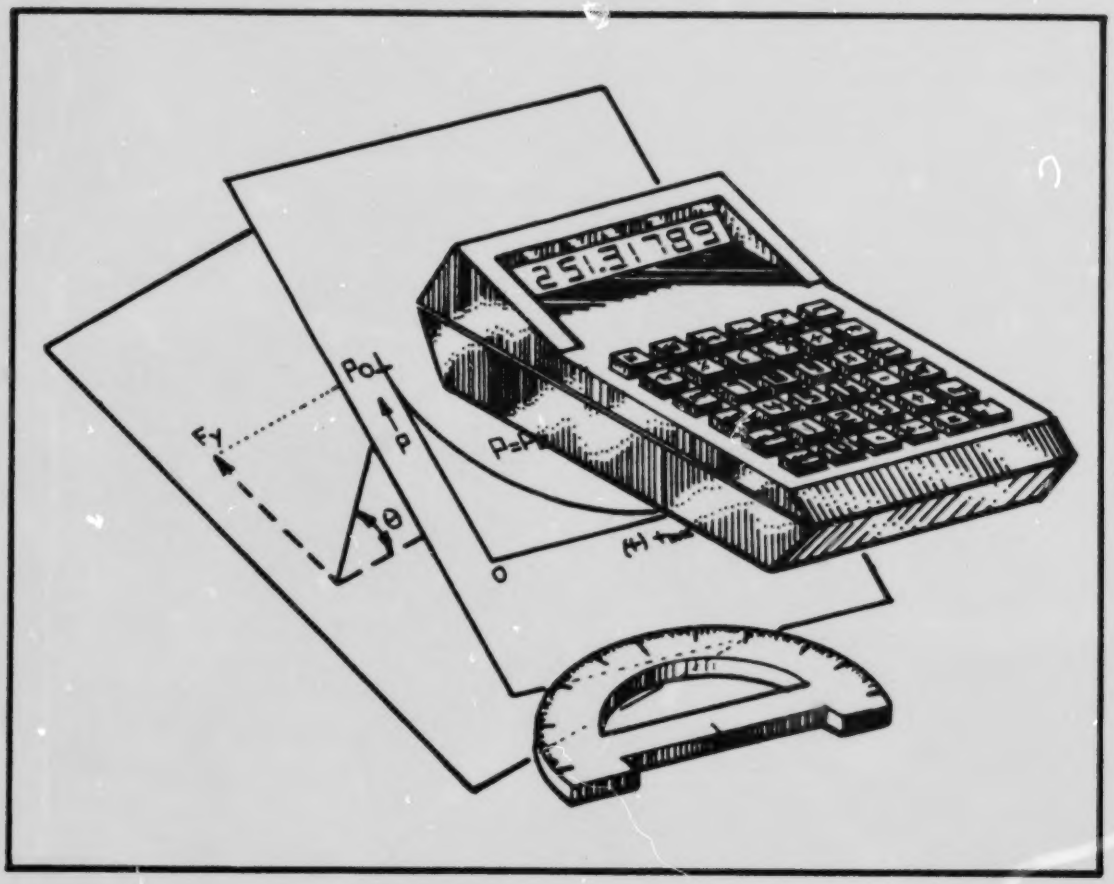
UNIFIED TECHNICAL CONCEPTS

MODULE S 1

FORMULA INTERPRETATION



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INTRODUCTION

Formulas and equations are a fundamental part of the language of science. Without them, a description of the relationships among physical quantities in any precise way would be almost impossible. Technicians must know how to use them correctly.

This module discusses how to understand and to manipulate the algebraic form of an equation and to apply that equation to a physical problem.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Define the following terms used in mathematical equations:
   a. Physical constant.
   b. Mathematical constant.
   c. Independent variable.
   d. Dependent variable.
2. Solve a simple algebraic equation or formula for any indicated variable, using combinations of mathematical operations of addition, subtraction, multiplication, division, raising to a power, and extracting a root.

> Example: Given the below equation, solve for A, B, C, D, and F:
>
> $$F^2 = \frac{AB}{C} - \sqrt{D}$$

3. For each of the following sets of quantities, write the basic mathematical formula stating their relationship, rearrange that equation to apply to a specific stated problem, and identify the elements of the resulting equation:
   a. Force, pressure, and area.
   b. Weight, mass, and acceleration of gravity.
   c. Voltage, current, and electrical resistance.

SUBJECT MATTER

Students of science soon learn that as their study of scientific principles progresses, more mathematics is encountered in the form of equations and formulas. Scientific facts and theories ultimately are expressed in equation form. Scientists, engineers, and technologists must be skilled in the mathematical principles used as tools of their work. To comprehend scientific principles, two major tasks are required. The first is to learn what the physical quantities mean and how they are measured. The second is to learn the relationship that exists among the physical quantities — usually a quantitative relationship expressed through symbols and an equation. If the first of these tasks were accomplished without the second, one could determine what the scientific principle was about, but not what it says about the behavior of nature. Accomplishment of the second without the first would be an exercise in pure mathematics, since the equations would not be interpretable. An understanding of the equations and of the meaning of the symbols which comprise them is the essence of understanding a scientific principle.

Equations and formulas are used to express the relationships among several physical quantities, as shown in Equation 1, which relates the concepts of force, pressure, and area.

$$\text{Force} = \text{Pressure} \times \text{Area} \qquad (1)$$

This equation provides a means of determining the value of one physical quantity if the numerical values of the other two are known. Frequently equations are written symbolically as in Equation 2.

$$\text{In Symbols:} \quad F = pA \qquad (2)$$

(In Words: Force = Pressure x Area)

where: F = Force (newtons or pounds).

p = Pressure (N/m^2 or lb/in^2).

A = Area (m^2 or in^2).

When two algebraic quantities are written together with nothing between them, they are to be multiplied together.

Symbolic equations often are used because they are compact, take less time to write, and are in a form for easy algebraic manipulation. Note that the symbols standing for the physical quantity often are the first letter of the physical quantity, although this is not essential. The symbol chosen to represent a physical quantity in an equation or formula is not important, but each symbol must be related clearly to a specific physical quantity.

A knowledge of the units of measure in which physical quantities are expressed is important. Some formulas in handbooks and technical material are often written so that only certain units of measure of the physical quantities involved will give the correct result when applying the formula. For example, the acceleration of gravity (symbolically "g") in the International System of Units (SI) has a value of 9.8 m/s^2 and in the English system, a value of 32 ft/s^2. Thus, the choice of "g" value used in Equation 3 depends upon the system of units chosen for the expression of weight and mass.

$$\underline{W}eight = \underline{M}ass \times acceleration\ of\ \underline{g}ravity \qquad (3)$$

In Symbols: $W = mg$

where: W = Weight (newtons or pounds).
m = Mass (kilograms or slugs).
g = Acceleration of gravity (9.8 m/s^2 or 32 f/s^2).

The scientist, engineer, and technician often must change formulas and equations to other desired forms by using the principles of algebra. When formulas are applied to different physical problems, they usually are changed to fit the problems. The relationship between the physical quantities is not changed. Only the algebraic form of that relationship is changed. To understand the purpose of this change of form, three concepts must be understood: the concept of a constant, an independent variable, and a dependent variable. In an equation, a constant has the same value in any form of the equation. An example of a constant concept is "g" in Equation 3 or 2π in Equation 4. A "physical constant" such as g depends upon the system of units employed; whereas, a "mathematical constant" such as 2π always has

the same value. The independent variable is a physical quantity whose value is known or measured in a physical problem. The dependent variable is the unknown in the physical problem, to be determined from the equation. The goal of changing the algebraic form of an equation is to isolate the dependent variable on one side of the equation and the constants and independent variables on the other, as shown below.

DESIRED FORM OF FORMULA FOR PHYSICAL PROBLEM
Dependent variable = Mathematical combinations of constants and independent variables.

As an example, the period of a simple pendulum is given by Equation 4.

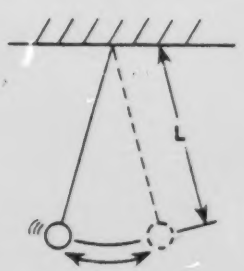
$$T = 2\pi\sqrt{\frac{\ell}{g}} \tag{4}$$

where: T = Period (seconds).
ℓ = Length (meters or feet).
g = Acceleration of gravity (m/s^2 or ft/s^2).

Examples A, B, and C demonstrate how Equation 4 is changed algebraically to be applied to three different problems.

EXAMPLE A: SOLVING FOR DEPENDENT VARIABLES.

Given: A pendulum exhibits simple harmonic motion and has a length of $\ell = 0.4$ m ($g = 9.8$ m/s^2).



Find: The period of oscillation (T).

Solution: Constants: $g = 9.8$ m/s^2, 2π.

Independent variable: ℓ.

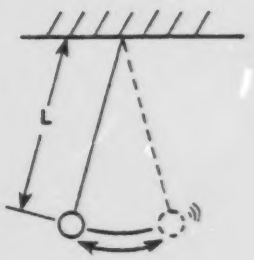
Dependent variable: T.

Desired form of Equation 4 applicable to this problem:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$ (No change is necessary.)

EXAMPLE B: SOLVING FOR DEPENDENT VARIABLES.

Given: A pendulum exhibits simple harmonic motion and has a length (ℓ) of 1 m and an oscillation period (T) of 2 seconds.



Find: The acceleration due to gravity.

Solution: Dependent variable: g.

Constants: 2π.

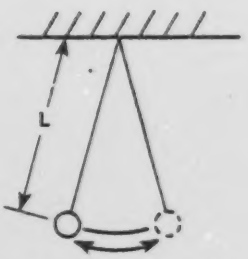
Independent variables: ℓ, T.

Desired form of Equation 4 applicable to this problem:

$$g = \frac{4\pi^2 \ell}{T^2}$$

EXAMPLE C: SOLVING FOR DEPENDENT VARIABLES.

Given: A pendulum exhibits simple harmonic motion and has a period (T) of 4 seconds. ($g = 9.8\ m/s^2$).



Find: The length (ℓ) of the pendulum.

Solution: Constants: 2π, $g = 9.8\ m/s^2$.

Independent variable: T.

Dependent variable: ℓ.

Desired form of Equation 4 applicable to this problem:

$$\ell = \frac{gT^2}{4\pi^2}$$

All of the different forms in the preceding examples can be derived from Equation 4 by using the rules of algebra. Notice that the desired dependent variable governs the equation form. Algebraically speaking, Equation 4 was solved for the desired dependent variables or particular problems.

SOLVING FORMULAS AND EQUATIONS

The ability to solve equations is an essential skill in the study of science, engineering, and mathematics. To "solve" an equation means to isolate a particular variable in an equation on one side of the equation by

itself. To accomplish this isolation, certain mathematical operations are performed on both sides of the equation. In the performance of these "operations," certain fundamental properties, or laws of equations, are used. These fundamental laws are known as "the associative law," "the commutative law," and "the distributive law." They are illustrated in Figure 1.

Law	Examples
Associative Law of Addition: $a + (b + c) = (a + b) + c$	$3 + (4 + 7) = (3 + 4) + 7$ $3 + 11 = 7 + 7$
Commutative Law of Addition: $a + b = b + a$	$5 + 8 = 8 + 5$
Associative Law of Multiplication: $(ab)c = a(bc)$	$(2 \times 3) \times 4 = 2 \times (3 \times 4)$ $6 \times 4 = 2 \times 12$
Commutative Law of Multiplication: $ab = ba$	$2 \times 4 = 4 \times 2$
Distributive Law of Multiplication: $(a + b)c = ac + bc$ where a, b, c are real numbers	$(2 + 3)6 = 2(6) + 3(6)$ $30 = 12 + 18$

Figure 1. Fundamental laws of equations.

The operations performed on the formulas yield equivalent equations. Any operation performed on one side of the equation always is performed on the other side. Many students neglect this principle. Care must be taken in these areas:

- Add or substract the same quantity to both sides of an equation.
- Multiply or divide both sides by the same quantity.
- Raise both sides to the same power.
- Extract the same root from both sides.

Frequently, a particular sequence of operations is required to solve an equation. The selection of the order in which the operations are performed is not a haphazard one. There are good reasons for a particular sequence of operations performed on both sides of the equation. Sometimes, there

may be more than one order of operation from which to choose, but each algebraically-valid ordering of a problem is equivalent and will yield the same solution. The solutions produced from different orders may appear in different forms, but always can be shown to be equivalent.

Several principles must be followed in order to determine which operations to accomplish first.

Step 1:

Simplify the equation by removing parentheses and other signs of grouping. Sometimes this removal is accomplished using the distributive law of algebra; sometimes, by multiplying or dividing both sides of the equation by the same factor. If fractions are involved, both sides of the equation should be multiplied by a common denominator. If radicals are involved, eliminate by raising each side of the equation to the appropriate power. Like terms should be collected if needed.

Step 2:

Add the same terms to both sides of the equation or subtract them from both sides. In this way, all terms involving the "solution" variable are transposed to one side of the equation, and all other terms are moved to the other side.

Step 3:

At this point, factoring may be indicated to obtain a dependent variable multiplied by a group of terms. In any event, at this point in the solution, there should be an equation similar to the general form found in Figure 2.

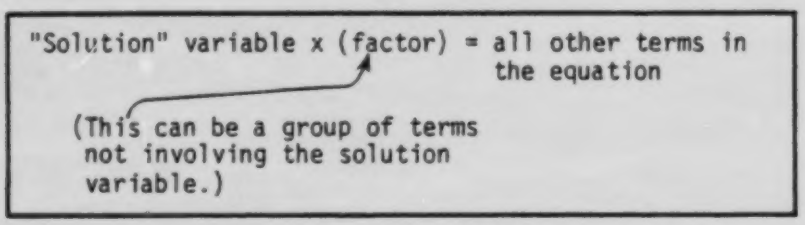

"Solution" variable x (factor) = all other terms in the equation
(This can be a group of terms not involving the solution variable.)

Figure 2. Factoring to obtain a dependent variable.

Step 4:

Both sides of the equation should be divided by the factor to obtain the solution variable alone on one side of the equation.

The proper performance of these maneuvers in isolating the dependent variable is grasped more easily through the study of specific examples. Examples D and E illustrate the preceding steps for solving equations.

EXAMPLE D: SOLVING AN EQUATION.

Given: $I = \frac{En}{R + nr}$

Find: E.

Solution: In solving for E, write the given equation:

Initial Equation:

$$I = \frac{En}{R + nr}$$

Step 1: Multiply both sides of the equation by the factor (R + nr):

$$(R + nr)\, I = (\cancel{R + nr})\left(\frac{En}{\cancel{R + nr}}\right)$$

$$(R + nr)\, I = En$$

Steps 2 and 3: No changes required.

Step 4: Divide both sides by n:

$$\frac{(R + nr)\, I}{n} = \frac{\cancel{En}}{\cancel{n}}$$

$$\frac{(R + nr)\, I}{n} = E \text{ , solution equation}$$

EXAMPLE E: SOLVING AN EQUATION.

Given: The equation $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

Find: p.

Solution:

Step 1: Multiply both sides of the equation by the least common denominator, fpq:

$$\cancel{f}pq\left(\frac{1}{\cancel{f}}\right) = f\cancel{p}q\left(\frac{1}{\cancel{p}}\right) + fp\cancel{q}\left(\frac{1}{\cancel{q}}\right)$$

Cancellation gives this result:

$$pq = fq + fp$$

Step 2: Subtract fp from both sides of the equation:

$$pq - fp = fq + \underbrace{fp - fp}_{0}$$

$$pq - fp = fq$$

Step 3: Factor p in the left-hand side:

$$p(q - f) = fq$$

solution variable — factor — all other terms of equation

Step 4: Divide both sides by the factor (q - f):

$$\frac{p\cancel{(q - f)}}{\cancel{(q - f)}} = \frac{fq}{(q - f)}$$

Solution Equation:

$$p = \frac{fq}{q - f}$$

For several reasons, students often fail to achieve an accurate solution. One common error is failure to perform the same operation to both sides of an equation. Other errors are misapplication of the distributive law by failure to distribute a factor completely through every term, thus getting the steps for solution out of order and leaving the unknown as part of the solution. There generally are several different procedures possible in isolating the dependent variable, although one method often may be more efficient.

APPLICATION OF FORMULAS TO PHYSICAL PROBLEMS

Many formulas and equations in physics are developed within the framework of certain basic assumptions. If a particular physical problem does not fit the assumptions on which the formula is based, then the formula cannot be used to give valid results for that problem. For example, earlier in this module three pendulum problems were used to illustrate how the same equation is applied in different algebraic forms. In each problem the pendulum was assumed to undergo simple harmonic motion; therefore, Equation 3 was valid only for that assumption. If a certain pendulum under study does

not exhibit simple harmonic motion, then Equation 3 cannot be used to describe correctly the motion of the pendulum in that situation. Assumptions upon which a formula or equation is based always must be considered in a physical-problem situation before a particular formula is applied to solve that problem.

SUMMARY

Mathematical equations are statements of equality. They involve two or more numbers or functions that may be known or unknown quantities. Formulas and equations contain the symbols required for the calculation of simple or complex mathematical problems. They often must be rearranged to solve for any unknown quantity with them. The three laws of numbers employed in the solving process are (1) the associative law, (2) the commutative law, and (3) the distributive law. If a change is made to one side of an equation, it must be made to the other side. The technician must be able to use formulas and equations in the solving of problems.

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Person, Russell V. Essentials of Mathematics. 3rd ed. New York: John Wiley and Sons, Inc., 1973.

REVIEW QUESTIONS

Complete the solutions to the following problems on separate paper. For problems 1-4, include the following steps:

A. Statement of problem.

B. General mathematical equation relating the elements of the problem.

C. Identification of each element in the equation including given values for the specific problem.

D. The steps necessary to rearrange the general equation to obtain the solution equation.

E. The numerical solution to the problem.

1. Find the force exerted on an area of one square foot by the atmosphere with a pressure of 14.7 psi (pounds per square inch).

   General equation: $F = pA$

2. An object weighs 100 Newtons. Determine its mass.

3. Solve the problem in Example C showing every step.

4. Ohm's law for electric circuits has the general form:

   Voltage = Current x Resistance

   $E = IR$

   where:

   E = Voltage (electrical potential) in volts (V).

   I = Current (charge flow rate) in Amperes (A).

   R = Electrical resistance in Ohms (Ω).

   Use Ohm's law to solve the following problem:

   An electrical heating element has a resistance of 10 Ω and is connected to a source of electrical potential at 100 V. How much current is flowing through the heating element?

In problems 5-10 solve each equation for the indicated variable. Write the equation and each step in the solution, showing cancellation steps. Rewrite a "clean" equation before proceeding to each step of the solution. Before each step, include a sentence that describes the operation applied to the equation. (Refer to Examples D and E for desired form.)

5. $E_p = mgh$ Solve for m.

6. $\frac{F_1}{F_2} = \frac{d_2}{d_1}$ Solve for d_1.

7. $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$ Solve for r_1.

8. $R = R_0 (1 + \alpha t)$ Solve for α.

9. $g = \frac{4\pi^2 \ell}{T^2}$ Solve for ℓ.

10. $T = 1/2 (P + p)S + B$ Solve for S.

11. What are the three laws of numbers? How many were used in this exercise?

The following equations are laws of definitions taken from science and engineering. Solve for the unknown variables indicated.

12. $V = IR$ Solve for R.

13. $a = \frac{v_2 - v_1}{t}$ Solve for v_2.

14. $v^2 = V_0^2 + 2as$ Solve for s.

15. $T = \sqrt{2 h/g}$ Solve for h.

16. $(m - ut)a = uv$ Solve for m.

17. $E_K = 1/2\ mv^2$ Solve for v.

18. $E_K = 1/2\ mv^2 + 1/2\ I\omega^2$ Solve for v.

19. $w^2 = \frac{2\mu h}{mr^2 + J}$ Solve for r.

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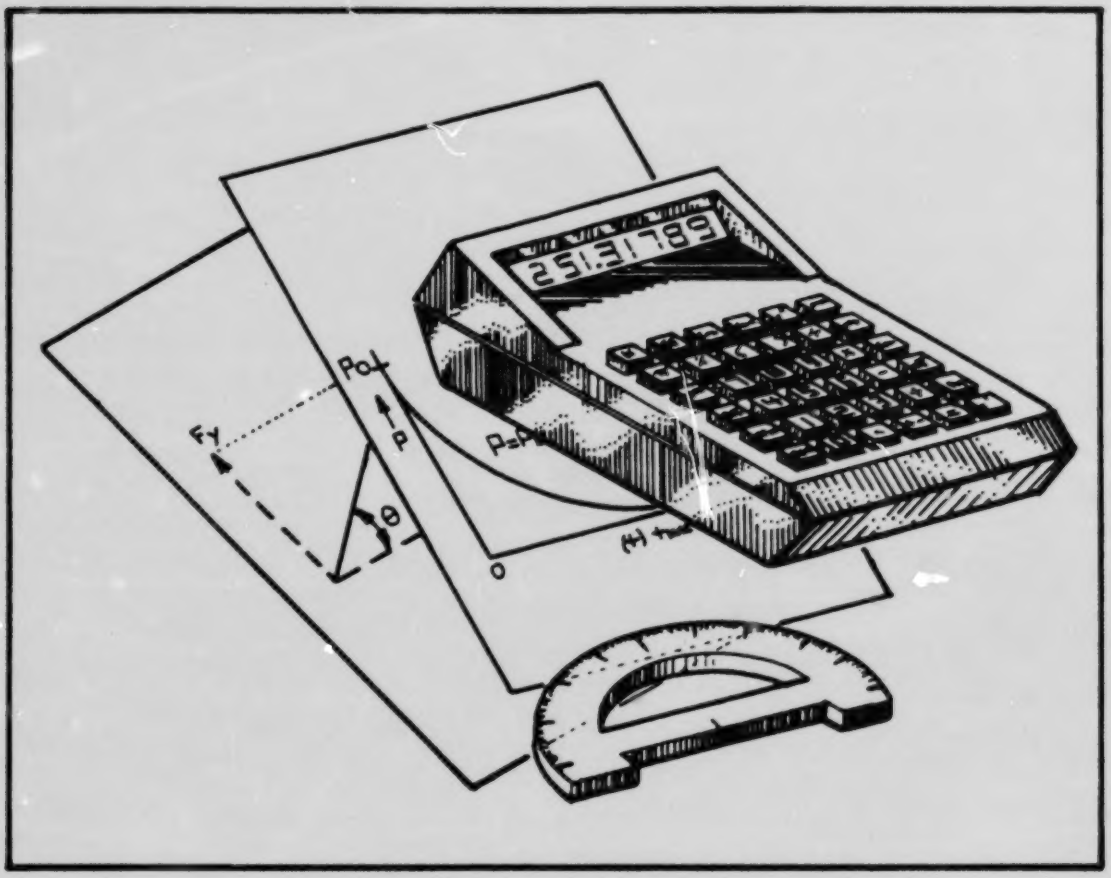
UNIFIED TECHNICAL CONCEPTS

MODULE S 2

DIMENSIONAL ANALYSIS



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INTRODUCTION

The method of determining the proper units for the solution in calculations is called "dimensional analysis." It is a useful procedure in studying and solving problems in physics and engineering because it is a check on whether the proper units were used when substituting values into equations. Careful attention paid to units will also reinforce the physics principles and will clarify the problem-solving process.

In this module, some advantages of dimensional analysis are presented, along with some of the techniques for handling units in calculations.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Distinguish between fundamental and derived units and give examples of each from SI and English units.
2. Explain the terms "dimensionless quantity" and "radian."
3. Given the steps in the solution of an equation, evaluate the correctness of the units in the solution.
4. Given an equation and the units of each term, determine the units of the solution.
5. Given the necessary equivalents, convert a physical quantity from one set of units to another using the ratio method.

SYSTEMS OF UNITS

Two systems of units commonly are used today in science and technology in this country. The systems are (1) the familiar English system and (2) the International Systems (SI) of units, which incorporates units of the older Metric system. Each of these systems contains fundamental, or basic, units that are defined in such a way as to be independent of all other units, and derived units that may be expressed and defined in terms of the fundamental units of the system.

FUNDAMENTAL UNITS

The fundamental units of the English system are the foot (length), the second (time), the pound (force), and the degree Fahrenheit (temperature). These fundamental units and the units derived from them are used to express all physical quantities in the English system. There are no English units for electrical energy systems.

The fundamental units in the SI system are the meter (length), the second (time), the kilogram (mass), the ampere (electric current), the Kelvin (temperature), and the candela (light intensity). This set of basic units and the units derived from them are employed to express all physical quantities in the SI system.

DERIVED UNITS

Units that are a comparison of two or more quantities and that require a multi-word label are referred to as "derived units." Miles per hour (mi/h) and pounds per square inch (lb/in^2) are examples of derived units. Many derived units are important enough to have special names, but they still are defined in terms of units.

In the SI system, the unit of mass is a basic unit called a kilogram (kg). The unit of force is a derived unit called a "newton" (N). One newton of force is the force necessary to produce an acceleration of one meter per second squared (m/s^2) when applied to a mass of one kilogram (kg). The following expression shows the definition of this derived unit in terms of basic units:

$$1\ N = \frac{kg \cdot m}{s^2} \qquad (1)$$

Equation 1 has the same form as the following equation known as Newton's Law:

$$F = ma \qquad (2)$$

where: F = Force (newtons).
m = Mass (kilograms).
a = Acceleration (meter per second squared).

In the English system, the unit of force (pound) is a basic unit, and the unit of mass is derived. One slug is the amount of mass that will experience an acceleration of one foot per second squared (ft/s^2) when acted upon by a one-pound (lb) force. The following equation is the definition of a slug in terms of basic units:

$$1\ slug = \frac{lb}{ft/s^2} = \frac{lb\ s^2}{ft} \qquad (3)$$

or

$$m = \frac{F}{a} \qquad (4)$$

Table 1 shows the units of length, time, mass, and force in both SI and English units.

TABLE 1. ENGLISH AND METRIC UNITS.

	Length	Time	Mass	Force
SI	meter (m)	second (s)	kilogram (kg)	newton* (N)
English	foot (ft)	second (s)	slug*	pound (lb)

*Derived units.

Other derived units with special names are the newton-meter ("joule"), the volt/amp ("ohm"), and the newton/meter^2 ("pascal"). All derived units may be written as an algebraic combination of basic units, as illustrated in Example A.

EXAMPLE A: DERIVED AND FUNDAMENTAL UNITS.

Given: a. 1 watt (W) = 1 joule/second (J/s).

b. 1 pascal (Pa) = 1 newton/meter^2 (N/m^2).

Find: Each of the above units in terms of basic SI units.

Solution: a. $1\ W = 1\ J/s$

$= 1\ \frac{N \cdot m}{s}$ since $1\ J = 1\ N \cdot m$

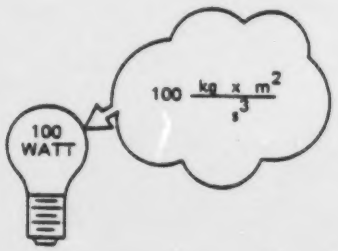
$= 1\ (\frac{kg \cdot m}{s^2})(\frac{m}{s})$ since $1\ N = 1\ \frac{kg \cdot m}{s^2}$

$1\ W = 1\ \frac{kg \cdot m^2}{s^3}$

b. $1\ Pa = 1\ N/m^2$

$= 1\ (\frac{kg \cdot m}{s^2})(\frac{1}{m^2})$

$1\ Pa = 1\ \frac{kg}{m \cdot s^2}$



Notice that in Example A the unit abbreviations are treated as algebraic variables. The processes of canceling and combining are accomplished according to the rules of algebra.

DIMENSIONLESS QUANTITIES

Dimensionless quantities often are encountered in calculations. Dimensionless quantities are treated simply as pure numbers without units. They usually are the result of a comparison, in the form of a ratio, of quantities having the same units. Because units are treated as algebraic quantities, any unit divided by itself simply cancels out.

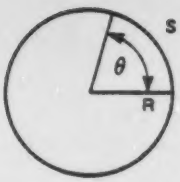


Figure 1. An angle of one radian.

A good example of this is a supplementary unit of angular measure in the SI system called a "radian" (rad). Figure 1 shows an angle of 1 rad (about 57.3°). An angle measured in radians is defined by Equation 5.

$$\theta = \frac{s}{r} \qquad (5)$$

where: θ = Angle in radians.

s = Length of arc subtended on a circle.

r = Radius of circle.

Clearly, since s and r are both in units of length, a radian is simply a ratio and carries no "real" units. The term "radian" is attached to the ratio to indicate that it expresses the size of an angle. "Radian" is dropped in all calculations because it is only a ratio.

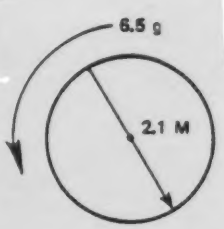
Some other dimensionless quantities are given in Table 2.

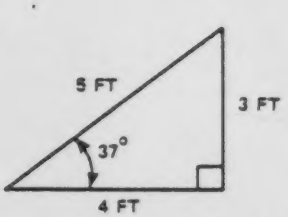
TABLE 2. DIMENSIONLESS QUANTITIES.

$$\pi = \frac{\text{circumference of circle}}{\text{diameter of circle}} = \frac{6.59\ \text{m}}{2.1\ \text{m}} = 3.14$$

$$\text{Tan } 37° = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{3\ \cancel{\text{ft}}}{4\ \cancel{\text{ft}}} = 0.7536$$

$$\text{Specfic gravity of iron} = \frac{\text{density of iron}}{\text{density of water}} = \frac{7900\ \frac{\cancel{\text{kg}}}{\text{m}}}{1000\ \frac{\cancel{\text{kg}}}{\text{m}}} = 7.9$$





CHECKING AN EQUATION DIMENSIONALLY

Operations performed with units are similar to the operations performed with numbers; therefore, if the units of an answer are not appropriate, some error has occured in the problem-solving process. One way to spot an erroneous equation is to check the dimensions of all of its terms. In any legitimate physical equation, dimensions of all terms on one side of an equation must be equal to dimensions of all terms on the other side of the equation.

Consider the equation $s = vt + (1/2)at^2$ where s represents distance, v represents speed (distance/time), t represents time, and a represents acceleration (distance/time^2). Substitution of metric units into the equation gives the following:

$$s = \left(\frac{\text{meters}}{s}\right)(s) + \left(\frac{\text{meters}}{s^2}\right)(s^2)$$

Like units cancel, resulting in these units:

$$s = \text{meters} + \text{meters} = \text{meters}$$

The equation is now termed a "dimensionally consistent" equation because s should have the unit "meters." A correct equation always is dimensionally consistent.

Sometimes equations are dimensionally correct, but the terms may be expressed in units incapable of addition or subtraction. For example, "1 meter + 40 centimeters" is dimensionally correct because it is a length added to a length; however, a conversion to one or the other of the units is necessary before the terms can be added [i.e., 1 meter + 0.4 meter = (1 + 0.4) meter = 1.4 meter].

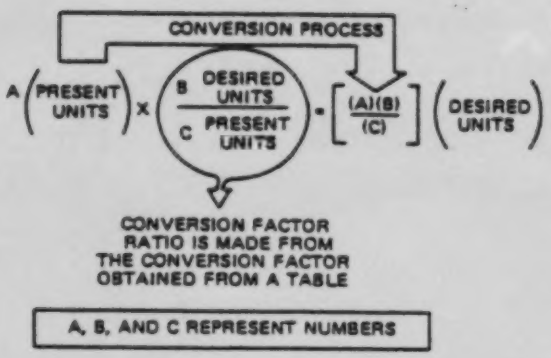

CONVERSION PROCESS
A (PRESENT UNITS) X (B DESIRED UNITS / C PRESENT UNITS) = [(A)(B)/(C)] (DESIRED UNITS)
CONVERSION FACTOR RATIO IS MADE FROM THE CONVERSION FACTOR OBTAINED FROM A TABLE
A, B, AND C REPRESENT NUMBERS

Figure 2. Units conversion process.

Dimensional analysis also is useful in solving unit-conversion problems. Figure 2 illustrates a one-step conversion process.

Examples of the conversion process are shown in Example B.

Notice that in Example B, arrangement of the conversion factor ratio is dictated by the units. In converting 6 ft to $\frac{?}{(12\text{ in} = 1\text{ ft})}$ in, suppose the conversion factor had been arranged incorrectly as follows:

$$6\text{ ft} = (6\text{ ft})\left(\frac{1\text{ ft}}{12\text{ in}}\right) = \frac{(6)(1)}{(12)} = 0.5\text{ ft}^2/\text{in}$$

The units of ft^2/in are certainly not the desired units.

EXAMPLE B: ONE-STEP UNIT CONVERSIONS.

Given: Units to be converted:

a. Convert 6 ft to ___?___ in (1 ft = 12 in).

b. Convert 12 in to ___?___ cm (1 in = 2.54 cm).

c. Convert 21 ℓ to ___?___ gal (ℓ = liter) (1 gal = 3.785 ℓ) Convert 21 ℓ to ___?___ gal (ℓ = 1 liter)

Find: Converted units.

Solution: (Use conversion process in Figure 1.)

a. $(6\text{ ft})\left(\frac{12\text{ in}}{1\text{ ft}}\right) = \left[\frac{(6)(12)}{(1)}\right]\text{ in} = 72\text{ in}$

b. $(12\text{ in})\left(\frac{2.54\text{ cm}}{1\text{ in}}\right) = \left[\frac{12\ (2.54)}{1}\right]\text{ cm} = 30.5\text{ cm}$

c. $(21\ \ell)\left(\frac{1\text{ gal}}{3.785\ \ell}\right) = 5.548\text{ gal}$

Use of a conversion ratio to change units makes possible the discovery of errors when they are made.

Conversion processes can be conducted in a step-by-step manner or in a "string" manner. The string method is particularly useful when converting derived quantities. An example of the string method is the problem of converting 30 miles/hour to ___?___ ft/s. Start by writing the original measurement:

$$30\ \frac{\text{mi}}{\text{h}}$$

Multiply by the conversion factor ratio necessary to convert miles to feet (5280 ft = 1 mile):

$$30\ \frac{\text{mi}}{\text{h}} = (30\ \frac{\text{mi}}{\text{h}})(\frac{5280\ \text{ft}}{1\ \text{mi}})$$

(arranged this way in order that the mile units will cancel)

Without multiplying the numbers, multiply this string by the conversion ratios necessary to convert hours to minutes (1 hour = 60 minutes) and minutes to seconds (1 minute = 60 seconds):

$$30\ \frac{\text{mi}}{\text{h}} = (30\ \frac{\cancel{\text{mi}}}{\cancel{\text{h}}})(\frac{5280\ \text{ft}}{1\ \cancel{\text{mi}}})(\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}})(\frac{1\ \cancel{\text{min}}}{60\ \text{s}})$$

(arranged this way in order that the hour and minute units will cancel)

Cancellation of the units reveals that the only units left are ft/s, which are the desired units. If cancellation does not result in the desired units, either an additional conversion factor is necessary or one of the factors included is in error. Once the desired units are assured, the numerical calculation is made:

$$30\ \frac{\text{mi}}{\text{h}} = \frac{(30)(5280)(1)(1)}{(1)(1)(60)(60)}\ \frac{\text{ft}}{\text{s}} = 44\ \frac{\text{ft}}{\text{s}}$$

Therefore: 30 miles/hour = 44 feet/second.

The same conversion factor can be used repeatedly as many times as indicated by the units. Example C shows 140 cubic inches being converted to cubic centimeters.

EXAMPLE C: CONVERTING UNITS.	
Given:	A volume (V) of 140 in^3.
Find:	This volume expressed in cm^3.
Solution:	$140\ in^3 = (140\ \cancel{in^3})(\frac{2.54\ cm}{1\ \cancel{in}})(\frac{2.54\ cm}{1\ \cancel{in}})(\frac{2.54\ cm}{1\ \cancel{in}})$
	$V = 140(2.54)^3\ cm^3$
	$V = 2294.2\ cm^3$

SUMMARY

The units on both sides of an equation must be equivalent. Dimensional analysis is a method of solving problems of, and locating errors based upon, this fact. When dimensional analysis is used to check the solution of a problem, the procedure is to determine if the units on each side of the equation are equivalent and appropriate. If they are, the solution may be correct. (Dimensional analysis will not reveal math errors.) If they are not, the solution is incorrect. A solution for distance in m/s, for example, cannot be correct.

Dimensional analysis also can be used as a key to the methods of solving problems; for example, since the units of velocity are distance units divided by time units, velocity is calculated by dividing distance by time.

Careful attention to the units involved in a problem can provide useful problem-solving information and can reduce errors.

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REVIEW QUESTIONS

MATERIALS

Pencil

Paper

1. Complete the following table:

Physical Quantity	English			SI		
	Name	Symbol	Basic Unit Equivalent	Name	Symbol	Basic Unit Equivalent
Length						
Time						
Mass						
Force						
Velocity						
Acceleration						

2. In the table in Problem 1 place an asterisk (*) following "name" column for each derived unit.

3. List below the names and symbols of units of length, time, mass, force, and velocity in both English and SI units. Identify which are basic units, and give the derived units in terms of basic units.

	English	SI
length	_____	_____
time	_____	_____
mass	_____	_____
force	_____	_____
velocity	_____	_____

Work the following problems on separate paper:

4. Given the equation $T = 2\pi\sqrt{\frac{I}{K}}$ where I has units: kg•m and K has units: $kg \cdot m/s^2$, what are the units of T?

5. In the study of fluids, an equation for Reynolds number (N_R) is given by $N_R = \rho vd/\eta$, where ρ has units of kg/m^3, v has units of m/s, d has units of m, and η has units of kg/m•s, what are the units of N_R?

6. Intensity of a sound wave is given by the equation, $I = \rho^2/2\rho_0 v$, where ρ has units of N/m^2, ρ_0 has units of kg/m^3 and v has units of m/s, what are the units of I?

7. A newton (N) = $kg \cdot m/s^2$, and a joule = $kg \cdot m^2/s^2$. What is the unit result of dividing a joule by a newton?

8. One volt (v) = $\frac{\text{joule (J)}}{\text{coulomb (C)}}$ and one ampere = $\frac{\text{coulomb (C)}}{\text{second (s)}}$ = C/s. What is the unit result of multipyling one volt by one ampere? (Hint: a J/s is equal to a watt.)

9. Calculate v_f, given this equation:

$$v_f^2 = v_0^2 + 2\,gs$$

where: v_0 = 20 cm/s.

s = 5 cm.

g = 980 cm/s^2.

10. In the following equations, substitute appropriate units for the symbols. Check for dimensional consistency, identifying the incorrect equations. (The symbol s is in meters; v_f, v_0, and v are in m/s; a is in m/s^2; t is in s.)

a. $s = \frac{v}{t}$

b. $a = s/t$

c. $v_f^2 = v_0 - 2\,as$

d. $s = v_0 + 1/2\,at^2$

11. Use the Conversion Factors Table on the next page to complete the following statements:

a. 35 knots = _____ furlongs/fortnight.

b. 45 miles per hour = _____ cm/shake.

c. 210 km/s = _____ ft/min.

d. 11 cm = _____ furlongs.

e. 8 mi^2 = _____ ft^2

Conversion Factor Table

1 mile = 5280 ft = 8 furlongs	1 h = 60 min
1 meter = 3.281 ft	1 min = 60 s
1 meter = 10^{10} angstroms (Å)	1 day = 24 h
1 meter = 10^{6} micron (μ)	1 fortnight = 14 days
1 fermi = 10^{-15} m	1 knot = 1.151 mi/h
1 in = 2.54 cm	1 shake = 10^{-8} s
12 in = 1 ft	

12. List the fundamental units of the English system of units and the SI system of units.

13. Write the equivalent expressions for the following units in terms of basic units:
    a. Slug.
    b. Newton.
    c. Joule.
    d. Watt.
    e. Newton-meter.

14. Given the equation $F = \frac{1}{2} kx^2$, where k has units of $kg/m \cdot s^2$ and x has units of m, determine the units of F.

15. Calculate mass density (ρ_m) of a right circular cylinder using the equation —

$$\rho_m = \frac{m}{\pi r^2 h}$$

where: ρ_m = Mass density.
m = Mass = 360 g.
r = Radius = 3 cm.
h = Height = 10 cm.

16. 1 volt = 1 J/C, 1 A = C/s, and 1 Ω = 1 V/A. Determine the equivalent of 1 Ω expressed in units of J, C, and s.

17. Using the table of conversion factors provided, work the following multiple-choice problems. Circle the correct answer.

1 Btu = 252 cal	1 bar = 10^5 pascals	60 min = 1 min
1 lb = 453.6 g	1 psi = 6895 pascals	(kg = kilogram)
1 C° = 1.8 F°	1 century = 100 yr	
1 C° = 1 K°	365 days = 1 yr	
1 cal = 4.184 J	24 h = 1 day	

a. The number of $\frac{\text{Btu}}{\text{lb/F°}}$ equivalent to 1.987 $\frac{\text{cal}}{\text{g/C°}}$ is –

1. 1.987. 2. 408,800. 6.438. 4. 3.576.

b. The number of $\frac{\text{joule}}{\text{kg/K}}$ equivalent to 1.987 $\frac{\text{cal}}{\text{g/C°}}$ is –

1. 8.314. 2. 8314. 3. 4619. 4. 1000.

c. The number of minutes in 1.0 microcentury is –

1. 0.0365. 2. 0.876. 3. 53. 4. 3180.

d. The production of synthetic diamonds from graphite requires a pressure of about 150 kilobars. This pressure expressed in psi is –

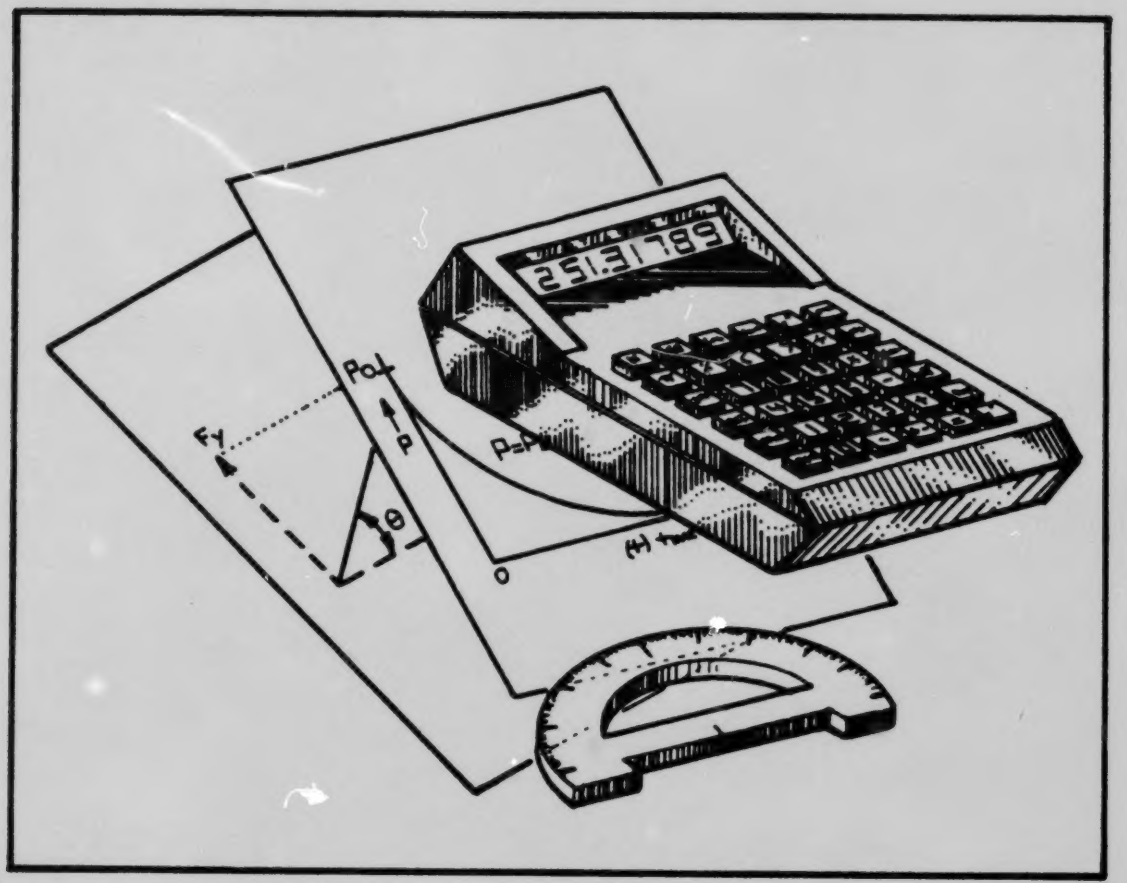
1. 2.2×10^6. 2. 1.5×10^5. 3. 22. 4. 1×10^{14}.

UTC

UNIFIED TECHNICAL CONCEPTS

MODULE S 3

INTERNATIONAL SYSTEM OF UNITS



SUPPLEMENTARY

CORD

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INTRODUCTION

Physics is an attempt to explain, in precise terms, the interactions of matter and energy. This attempt requires a consistent system of units for measuring and describing all the properties of matter and energy. The International System of Units (abbreviated SI for the French name Le Système International d' Unités) is the accepted system of units for science and technology throughout the world. This lesson discusses the units within the SI system, prefixes used with these units, proper procedure for writing SI units, and conversions from SI units to English units.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Name the seven basic SI units, and give the symbol and quantity measured for each.
2. Name the two supplementary SI units, and give the symbol and quantity measured for each.
3. List the derived SI unit and its symbol for each of the following quantities:
   a. Frequency.
   b. Force.
   c. Energy.
   d. Power.
   e. Electrical charge.
   f. Electrical potential.
   g. Electrical resistance.
4. State the name and symbol of the prefix for each of the following multiples:
   a. 10^{12}.
   b. 10^{9}.
   c. 10^{6}.
   d. 10^{3}.

e. 10^{-2}.

f. 10^{-3}.

g. 10^{-6}.

h. 10^{-9}.

i. 10^{-12}.

j. 10^{-15}.

k. 10^{-18}.

5. Given quantities expressed in SI units, use prefixes to write an expression for the same quantity with the numerical values between 0.0 and 1000.
6. Write, in the accepted notation, expressions indicating the products, quotients, and reciprocals of given SI units.
7. Name one non-SI unit that may be used with SI units for measuring the following quantities:
   a. Time.
   b. Plane angle.
   c. Volume.
   d. Mass.
8. Convert lengths in feet and inches to the equivalent length in SI units.
9. Given a mass in kilograms, calculate the weight in pounds.

ORIGIN OF THE INTERNATIONAL SYSTEM OF UNITS

The International System of Units, a concise set of units allowing measurement of all physical quantities, was adopted by the International Organization of Standardization in 1972. The basis of the SI system is the older metric system. This system was chosen because it is a decimal system; that is, each unit for a particular quantity is related to all other units for that quantity by a multiple of ten.

All standards of measurement are arbitrary. Their virtue lies in the fact that they are useful, reproducible, and accepted by those who use them. The original meter was intended to be one ten-millionth the distance from the north pole to the equator, along a line passing through Paris. This distance was indicated by two fine lines scribed on gold plugs in an X-shaped platinum-iridium bar. When this bar is at 0°C (the temperature of melting ice), the distance between the marks is defined as one meter. This distance actually is different from the originally intended distance by 0.023%; but this is unimportant, because the standard is arbitrary.

The standard kilogram originally was intended to be the mass of 1000 cm^3 of pure water at 4°C. Once again, this measurement is not exactly accurate. One kilogram is defined to be equal to the mass of a platinum-iridium cylinder called the "standard kilogram," and carefully preserved. The second was chosen arbitrarily as 1/86,400th the duration of the average solar day. Modern definitions of the meter and of the second are more precise; they are given later in this module.

Several other quantities besides length, mass, and time must be measured for a complete description of the physical world. There are many ways of defining basic units. All electrical units, for example, could be defined in terms of voltage, current, or charge. In the SI system, current has been chosen arbitrarily as the fundamental quantity. Once a system of base units has been established, all other necessary units can be derived from them. In any measurement system, the base units are arbitrary.

UNITS IN THE SI SYSTEM

There are three classifications of units in the SI system. The base units are given in Table 1, along with their symbols and the fundamental quantities they are used to measure. The definition of each base unit is completely arbitrary and is independent of all other base units. Their exact definitions appear later in this module.

TABLE 1. SI BASE UNITS.

Quantity	Name of Base SI Unit	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Table 2 lists the two supplementary units of the SI system. These units are used to measure plane and solid angles and do not meet the exact criteria for either base or derived units.

TABLE 2. SI SUPPLEMENTARY UNITS.

Quantity	Name of Supplementary SI Unit	Symbol
plane angle	radian	rad
solid angle	steradian	sr

A large number of units can be derived from the base and supplementary SI units. Table 3 lists the derived units of the SI system that have special names. Many other common derived units do not have specific names; examples are velocity, measured in meters per second (m/s), and volume, measured in cubic centimeters (cm^3).

TABLE 3. SI DERIVED UNITS FROM NAMES.

Quantity	Name of Derived SI Unit	Symbol	Expressed in Terms of Base or Supplementary SI Units or in Terms of other Derived SI Units
frequency	hertz	Hz	$1\ Hz = 1\ s^{-1}$
force	newton	N	$1\ N = 1\ kg \cdot m/s^2$
pressure, stress	pascal	Pa	$1\ Pa = 1\ N/m^2$
energy, work, quantity of heat	joule	J	$1\ J = N \cdot m$
power	watt	W	$1\ W = 1\ J/s$
electric charge, quantity of electricity	coulomb	C	$1\ C = 1\ A \cdot s$
electrical potential, potential difference, tension, electromotive force	volt	V	$1\ V = 1\ J/C$
electric capacitance	farad	F	$1\ F = 1\ C/V$
electric resistance	ohm	Ω	$1\ \Omega = 1\ V/A$
electric conductance	siemens	S	$1\ S = 1\ \Omega^{-1}$
flux of magnetic induction, magnetic flux	weber	Wb	$1\ Wb = 1\ V \cdot s$
magnetic flux density, magnetic induction	tesla	T	$1\ T = 1\ Wb/m^2$
inductance	henry	H	$1\ H = 1\ Wb/A$
luminous flux	lumen	lm	$1\ lm = 1\ cd \cdot sr$
illuminance	lux	lx	$1\ lx = 1\ lm/m^2$

The prefixes given in Table 4 are an important part of the SI system. They allow any numerical value to be written as a number between 0.1 and 1000, as shown, for example, in Table 4. This may at first seem a trivial convenience, but it actually is one of the most important characteristics of the SI system. The most important prefixes in Table 4 are indicated by an asterisk. Centi (10^{-2}) is used primarily with meter to form the useful centimeter. Table 5 shows the use of these prefixes. Notice that in some cases there are two acceptable statements of the same quantity (0.325 mm and 325 μm). The kilogram is a special case. Because the name of this base unit of mass contains the name of SI prefix "kilo," the names of the decimal multiples and submultiples of the unit of mass are formed by adding the prefixes to the word "gram." Thus, 10^{-6} kg is written as milligram (mg) instead of microkilogram (μkg).

TABLE 4. SI PREFIXES.

Factor by Which the Unit is Multipled	Prefix	
	Name	Symbol
10^{12}	tera	T
* 10^{9}	giga	G
* 10^{6}	mega	M
* 10^{3}	kilo	k
10^{2}	hecto	h
10	deca	da
10^{-1}	deci	d
* 10^{-2}	centi	c
* 10^{-3}	milli	m
* 10^{-6}	micro	μ
* 10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

TABLE 5. USE OF SI PREFIXES.

Quantity Expressed in Base or Dervied Units	Quantity Restated in Base or Dervied Units	Quantity Expressed with Prefix and Base of Dervied Unit
12000 N	12×10^{3} N	12 kN
0.000325 m	0.325×10^{-3} m or 325×10^{-6}	0.325 mm or 325 µm
1401 V	1.401×10^{3} V	1.401 kV
34200 W	34.2×10^{3} W	34.2 kW
1.87×10^{-7} S	0.187×10^{-6} S or 187×10^{-9} S	0.187 µs 187 ns
3.28×10^{7} J	32.8×10^{6} J	32.8 MJ
0.95 kg	950×10^{-3} kg	950 g

RULES FOR WRITING SI UNITS

The following are rules for writing SI units:

- The unit symbols are written in lower case letters, except that the first letter is written in upper case when the unit is derived from a proper name.

  Examples: m — meter A — ampere
  s — second Wb — weber

- Compound units formed by the multiplication of two or more units may be written in one of the following ways:

  N•m N.m Nm

  The first of these is most common and should be used to eliminate possible confusion. (Notice that mN could be read as millinewton or meter times newton if the dot indicating multiplication is not used consistently. Such ambiguity must be avoided.)

- Compound units formed by the division of one unit by another are written in the following ways:

$$m/s \qquad \frac{m}{s} \qquad m \cdot s^{-1}$$

  The notation s^{-1} is the same as 1/s and indicates the reciprocal of the base or derived unit.
- In no case should more than one solidus (/ as in m/s) on the same line be included in such a combination unless parentheses are included to avoid all ambiguity. Power of units may also be used:

  Example: unacceptable — m/s/s
  acceptable — (m/s)/s m/s^2 $m \cdot s^{-2}$
- Only one prefix may be used when forming a multiple or submultiple of a base or derived unit.

  Example: unacceptable — mμm (millimicrometer)
  acceptable — nm (nanometer)

Example A shows several improper expressions written in SI units and a better statement of each quantity. Such "improper" expressions are in common use and some will probably remain so, because they are convenient in some calculations.

EXAMPLE A: WRITING QUANTITIES WITH SI UNITS.	
Given:	The following table containing quantities expressed improperly in SI units.
Find:	Proper expression for each quantity.

Solution:

Quantity Measured	Improper Expression	Better Expression
length	2480 m	2.48 km
mass	0.00032 kg	0.32 g or 320 mg
time	6.72×10^{-5} s	67.2 μs
velocity	0.025 m/s	2.5 cm/s or 25 mm/s
energy	5500 mJ	5.5 J
acceleration	4.3×10^{3} m/s/s	4.3 km/s²
power	2.5 kmW	2.5 GW

OTHER UNITS USED WITH SI UNITS

Certain units outside the SI system frequently are used with SI units because of their practical importance (Table 6) or because of the needs of specialized fields (Table 7). In a limited number of cases, compound units are formed from these units and SI units; for example, kg/h and km/h. Such units are not strictly part of the SI system.

TABLE 6. NON-SI UNITS USED WITH SI UNITS BECAUSE OF PRACTICAL IMPORTANCE.

Quantity	Name of Unit	Unit Symbol	Definition
time	minute	min	1 min = 60 s
	hour	h	1 h = 60 min
	day	d	1 d = 24 h
plane angle	degree	°	1° = (π/180) rad
	minute	'	1' = (1/60)°
	second	"	1" = (1/60)'
volume	liter	l	1 l = (10 cm)³
mass	tonne	t	1 t = 10^{3} kg

TABLE 7. SPECIAL UNITS USED WITH SI UNITS.

Quantity	Name of Unit	Unit Symbol	Definition
energy	electron-volt	eV	1 electron-volt is the kinetic energy acquired by an electron in passing through potential difference of 1 volt in vacuum; 1 eV = 1.60219×10^{-19} J
mass of an atom	atomic mass unit	u	1 (unified) atomic mass unit is equal to the fraction 1/12 the mass of an atom of the nuclide C^{12}; 1 u = 1.66053×10^{-27} kg
length	astronomic unit	*AU	1 AU = $149{,}600 \times 10^{6}$ m (System of astronomic constants, 1964)
	parsec	pc	1 parsec is the distance at which 1 astronomic unit subtends an angle of 1 second of arc; 1 pc = 206,265 AU = 3.0837×10^{16} m
pressure of fluid	**bar	bar	1 bar = 10^{5} Pa

*This unit has no international symbol; AU is the abbreviation of the English name; the abbreviation of the French name is UA.

**The bar is not mentioned by CIPM in this group of units; in many countries, however, there are special requirements for this unit.

SI is not the only system of units in use today. Others, such as the English system, are frequently used. Consequently, quantities expressed in SI units often must be converted to equivalent expressions in English units. Table 8 lists some of the more useful conversion factors. The use of these conversion factors is illustrated in Example B.

TABLE 8. CONVERSION TO SI UNITS.

Quantity	Non SI Unit	SI Equivalent
length	1 inch (in)	2.54 cm
length	1 yard (yd)	0.9144 m
force	1 pound (lb)	4.445 N
power	1 horsepower (hp)	746 W
energy	1 calorie (cal)	4.186 J

EXAMPLE B: CONVERSION TO SI UNITS.

Given: The following quantities:

a. 12 in

b. 0.25 hp

c. 250 cal

Find: Equivalent expressions in SI units.

Solution:

a. $(12\ \cancel{in})\left(\frac{2.54\ cm}{1\ \cancel{in}}\right) = 30.48\ cm$

b. $(0.25\ \cancel{hp})\left(\frac{746\ W}{1\ \cancel{hp}}\right) = 186.5\ W$

c. $(250\ \cancel{cal})\left(\frac{4.186\ J}{\cancel{cal}}\right) = 1046.5\ J = 1.0465\ kJ$

In the SI system, the base unit of mass is the kilogram and the unit of force (N) is derived. The English system uses a base unit of force called the "pound" and a less-familiar derived mass unit called a "slug." There can be no conversion of pounds to kilograms, because the two units do not measure the same quantity. (The two systems were developed from two different arbitrary starting points.) One should remember, however, that one kilogram weighs 2.2 pounds on the earth's surface, as illustrated in Example C.

EXAMPLE C: WEIGHT OF MASS IN KILOGRAMS.	
Given:	Mass of 84 kg.
Find:	Weight in pounds.
Solution:	1 kg weight 2.2 lb
	$(84\ \cancel{kg})\ (\frac{2.2\ lb}{\cancel{kg}}) = 184.8\ lb$

DEFINITIONS OF UNITS

A knowledge of the exact definition of SI units is not essential to the use of these units. However, it is interesting to note the precision with which they are defined and their arbitrary nature. Some of the definitions follow:

BASE UNITS

- Meter — The meter is the length equal to 1,650,763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom.
- Kilogram — The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.
- Second — The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.
- Ampere — The ampere is that constant electric current which, if maintained in two straight, parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.
- Kelvin — The Kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
- Mole — The mole is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12. When the mole is used, the elementary entities must be

specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.

- Candela — The candela is the luminous intensity, in the perpendicular direction, of a surface of 1/600,000 square meters of a black body, at the temperature of freezing platinum, under a pressure of 101,325 newtons per square meter.

SUPPLEMENTARY UNITS

- Radian — The radian is the plane angle between two radii of a circle which cut off on the circumference an arc equal in length to the radius.
- Steradian — The steradian is the solid angle which, having its vertex at the center of a sphere, cuts off an area of the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere.

SUMMARY

The International System of Units (SI) is now the accepted system of measurement units throughout the world. Based largely upon the Metric System, the SI System is most important in technology. There are base units, units derived from names, and prefixes. Other units not a part of the SI System are used with it; these units include the minute, the hour, and the day.

The seven basic SI units are the meter (symbol m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol), and candela (cd). The radian (rad) and steradian (sr) are supplementary units. Derived units include the hertz (Hz), newton (N), joule (J), watt (W), coulomb (C), volt (V), farad (F), ohm (Ω), weber (Wb), tesla (T), henry (H), lumen (lm), lux (lx), pascal (Pa), and siemens (S).

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REVIEW QUESTIONS

1. Use the necessary prefixes, and write the following quantities in SI units with a numerical value between 0.1 and 1000:

   a. 2.8×10^4 m = _____

   b. 2.8×10^4 nm = _____

   c. 2.8×10^4 mm = _____

   d. 350,000 Hz = _____

   e. 3.5×10^9 Hz = _____

   f. 1.45×10^{-8} s = _____

   g. 627,000,000 Ω = _____

   h. 0.0015 A = _____

   i. 0.015 cm = _____

   j. 8.4×10^8 W = _____

   k. 2.3×10^{-6} kg = _____

   l. 0.04 N = _____

2. Find the following values, and express the results in proper SI notation:

   a. If t = 3 s, then $\frac{1}{t}$ = _____

   b. If L = 0.15 cm, then $\frac{1}{L}$ = _____

   c. If f = 60 Hz = 60 s^{-1}, then $\frac{1}{F}$ = _____

3. Complete the following chart of SI base units.

Symbol	Name	Quantity Measured
s		
kg		
mol		
A		
cd		
K		
m		

4. List the quantity measured and the symbol for the following SI supplementary units:
   a. Radian.
   b. Steradian.
5. Complete the following chart of SI derived units.

Symbol	Name	Quantity Measured
W		
J		
Hz		
N		
V		
Ω		
C		

6. State the symbol and the multiple of ten indicated by each of these prefixes:
   a. Milli.
   b. Micro.
   c. Centi.
   d. Mega.
   e. Giga.
   f. Kilo.
   g. Nano.
7. Write the following quantities in proper SI notation:
   a. 4,876,000 Ω.
   b. 0.0027 cm.
   c. 1,440 mW.
   d. 5.2×10^{4} g.
   e. 0.055 ms.

8. Name a non-SI unit that may be used with SI units for measuring the following quantities:
   a. Time.
   b. Volume.
   c. Plane angle.
   d. Mass.
9. Convert these lengths to SI units:
   a. 16 ft.
   b. 27.5 in.
   c. 0.025 in.
10. Calculate the weight in pounds of the following masses:
   a. 15 kg.
   b. 350 g.

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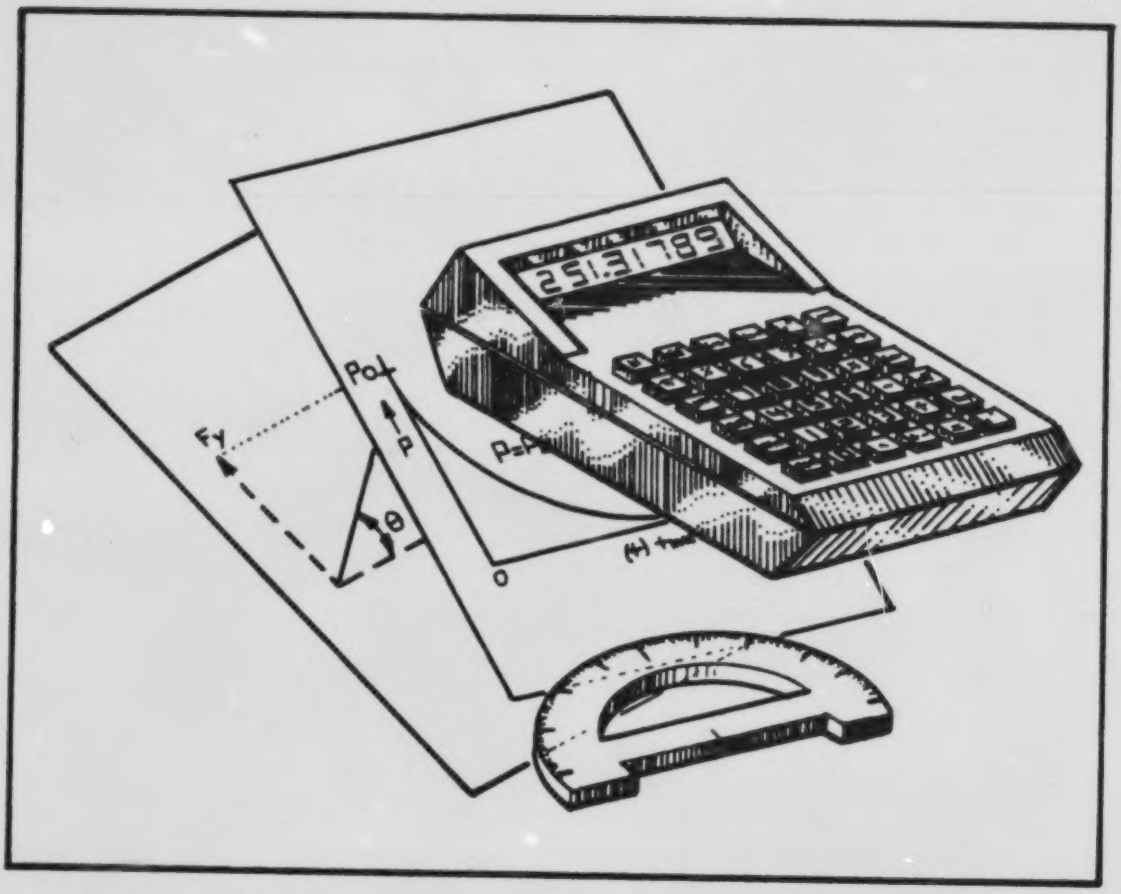
UNIFIED TECHNICAL CONCEPTS

MODULE S 4

ANGLES AND TRIANGLES



SUPPLEMENTARY

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INTRODUCTION

The use of angles and triangles in technical work is extensive and very important to the study of physics, electronics, lasers/electro-optics instrumentation, civil engineering, mapping, construction, biomedical equipment, electromechanical, electrical power, and many other technologies. This module introduces angles and their measurement to the extent necessary to understand their use in triangles of several different types. Similar triangles, right triangles, and Pythagorean Theorem are discussed. Trigonometric ratios of sine, cosine, and tangent are presented, as well as applications of these ratios to right angles, vector addition, and resolution of vectors into right-angle components.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Identify the following terms:
   a. Angle.
   b. Right angle.
   c. Straight angle.
   d. Obtuse angle.
   e. Triangle.
   f. Right triangle.
   g. Acute triangle.
   h. Obtuse triangle.
   i. Similar triangles.
2. Measure an angle, given a protractor.
3. Define these terms:
   a. Degree.
   b. Radian.
   c. Sine of an angle.
   d. Cosine of an angle.
   e. Tangent of an angle.

4. Given two similar right triangles, a ruler, and a protractor, verify:
   a. That the ratio of corresponding sides of similar triangles is equal.
   b. That all angles of a triangle add to 180°.
5. Given a right triangle with two known sides, apply the Pythagorean theorem to determine the third side.
6. Given stated problems involving right-triangle configurations, solve the problem for the unknown side or angle using the definition of sine, cosine, or tangent, and the Pythagorean theorem.
7. Given a vector, resolve it into two mutually-perpendicular component vectors.

ANGLES (THE SUBJECT OF TRIGONOMETRY)

Two straight lines that meet at a point form an angle. Figure 1 shows the two straight lines AB and AC meeting at point A. Point A is called the "vertex of the angle." Often in diagrams involving angles, letters are placed near the vertex to identify the angle. Also the symbol ∠ often is used instead of the work "angle." The size of the angle is determined by the amount of rotation required if one side is rotated about the vertex into the other side, as shown in Figure 1. The measure of this rotation is expressed in a unit called a "degree." A degree is defined as 1/360 a circle; a circle includes 360 degrees. The symbol ° placed to the right of and above a number usually is employed instead of the word "degree." Sixty degrees is written 60°.

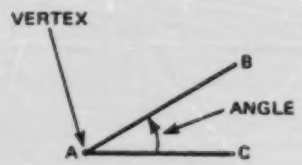


Figure 1. Components or an angle.

Certain angles measured in degrees are given names. Figure 2 illustrates these angles. Two lines perpendicular to one another form a right angle of 90°; two lines forming a straight line make a straight angle of 180°. An angle less than 90° is called an "acute angle." An angle greater than 90° but less than 180° is termed an "obtuse angle."

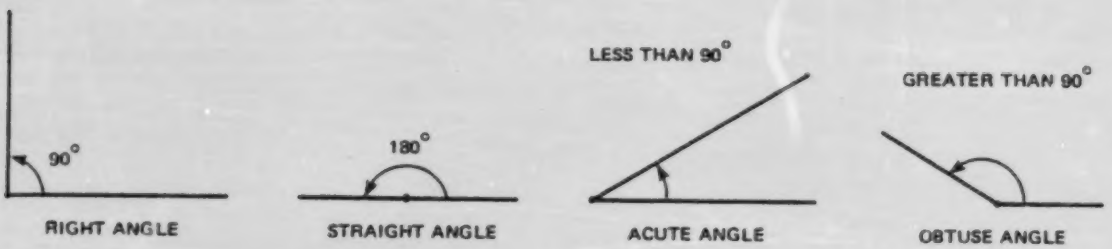


Figure 2. Types of angles.

UNITS OF ANGULAR MEASURE

As mentioned previously, the degree is a common unit of angular measurement. However, it is a small unit of angular measurement. Figure 3 illustrates the size of one degree. The two lines forming the sides of the 1°

Figure 3. A one-degree angle.

angle in Figure 3 would have a separation of 1 inch when measured at 57 inches from the apex of the angle. In technical work, angles frequently are measured in a unit of larger size, called the "radian." The radian is defined in terms of a piece of a circle. A radian is the angle subtended by the arc of a circle equal in length to the radius as shown in Figure 4. The angle θ produced at the center of the circle would be described as one radian. The angle θ would be equivalent to 57.2958 degrees. With an electronic calculator, one can verify that one radian expressed in degrees multiplied by π(3.1415927) will give a result of 180 degrees. There are, therefore, π radians in a semicircle (180 degrees). This fact is often used in converting from degrees to radians or from radians to degrees, as indicated in Example A.

Figure 4. Definition of radian.

EXAMPLE A: CONVERSIONS OF RADIANS AND DEGREES.		
Given:	Angles of 20° and 2.5 rad.	
Find:	a. The equivalent of 20° in radians.	
	b. The equivalent of 2.5 rad in degrees.	
Solution:	To convert —	multiply by —
	degrees to radians	π/180
	radians to degrees	180/π
	a. $20° \times (\pi/180) = 0.35$ rad	
	b. $2.5 \times (180/\pi) = 143°$	

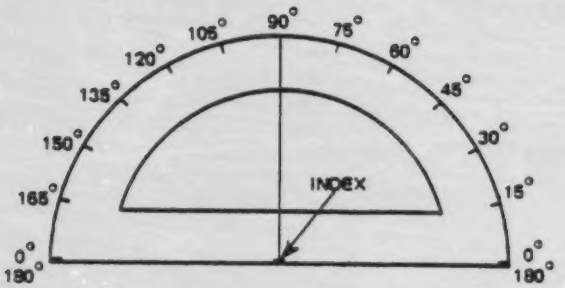


Figure 5. Components of a protractor.

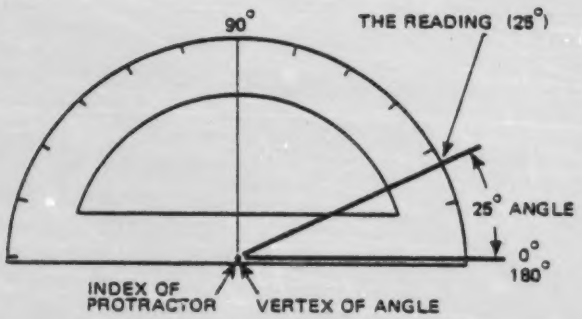


Figure 6. Measuring a 25° angle.

MEASUREMENT OF ANGLES

The protractor, an instrument used in the measurement of angles, consists of a semicircular scale of convenient size (Figure 5). This scale is divided into degrees, and the divisions of the scale are numbered from 0° to 180°, beginning at each end. This method of numbering enables one to measure an angle from either end of the protractor.

To measure an angle with a protractor, align the protractor with one side of the angle to be measured so that the vertex of the angle and the index of the protractor coincide. (See Figure 6). The reading on the scale where the second side of the angle crosses the scale part of the protractor is the measure of the angle in degrees. Figure 6 illustrates the measurement of a 25-degree angle.

TRIANGLES

A polygon is a plane surface bounded by any number of straight lines called "sides." A triangle is a polygon of three sides, containing three angles, as shown in Figure 7. Applications of triangles are very common in industrial work such as bridge construction and architecture. Triangles often are classified by the types of angles that they contain. Three of the angles used for classification were illustrated in Figure 2 of this module.

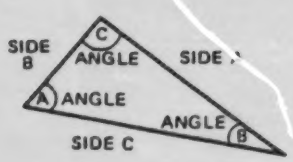


Figure 7. Components of a triangle.

An obtuse triangle contains one obtuse angle (greater than 90°) and two acute angles. A right triangle contains one right angle (equal to 90°) and two acute angles. An acute triangle contains only angles less than 90°. An example of each of these three types of triangles is given in Figure 8.

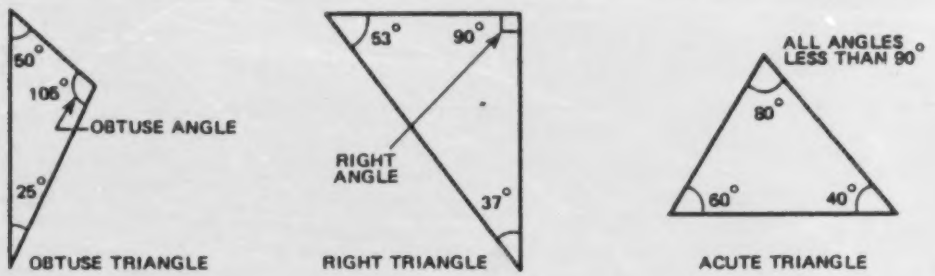


Figure 8. Types of triangles.

One fundamental principle that applies to all triangles is the angle-sum principle:

THE SUM OF THE THREE ANGLES OF ANY TRIANGLE IS 180°

This fact can be verified if the angles of each triangle illustrated in Figure 8 are added. The angle-sum principle is particularly useful when two angles of a triangle are known and the third angle is to be found, as in Example B.

EXAMPLE B: FINDING THE UNKNOWN ANGLE OF A TRIANGLE.

Given: A triangle contains angles of 70° and 45°.

Find: The third angle of the triangle.

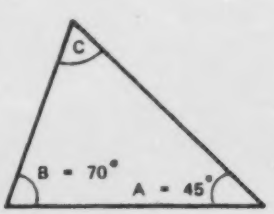
Solution: The sum of the angles is 180°.

$$\angle A + \angle B + \angle C = 180°$$

$$\angle C = 180° - A - B$$

$$\angle C = 180° - 70° - 45°$$

$$\angle C = 65°$$



SIMILAR TRIANGLES

When the angles of one triangle equal the angles of another triangle, the two triangles are said to be "similar." The triangles may be of different size, yet their corresponding angles are equal. Two similar triangles are illustrated in Figure 9. Corresponding sides of similar triangles are

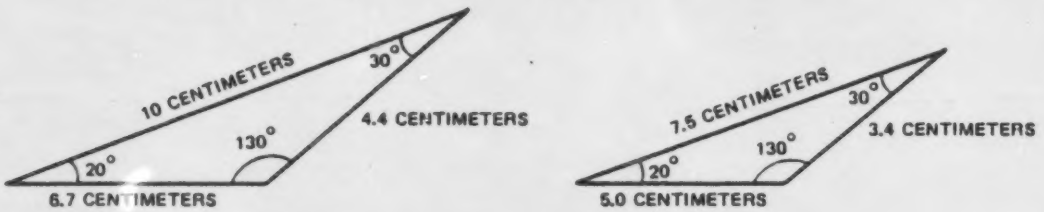


Figure 9. Similar triangles.

located across from the same angles of each triangle. In Figure 9, the 10-cm side in the large triangle and the 7.5-cm side in the smaller triangle are corresponding sides, both across the triangles from the 130° angle. Likewise, the 6.7-cm and 5.0-cm sides are corresponding, as are the 4.4-cm and 3.4-cm sides. Corresponding sides of similar triangles must have the same ratio. This ratio tells how much larger one triangle is than the other in terms of its linear dimensions. For the two triangles in Figure 9, the ratios of corresponding sides can be computed as follows:

$$\frac{10 \text{ cm}}{7.5 \text{ cm}} = 1.3$$

$$\frac{6.7 \text{ cm}}{5.0 \text{ cm}} = 1.3$$

$$\frac{4.4 \text{ cm}}{3.4 \text{ cm}} = 1.3$$

These identical ratios show that the linear dimensions of the larger triangle are 1.3 times larger than the corresponding dimensions of the smaller triangle.

RIGHT TRIANGLES

The most common use of ratios and proportions involving triangles is related to right triangles. A right triangle contains one 90° (right) angle. The side across the triangle from the 90° angle is called the "hypotenuse" and is the largest side of the triangle. The other two shorter sides sometimes are called "legs." A sample right triangle is shown in Figure 10. The relationship between the legs and hypotenuse of a right triangle is very important.

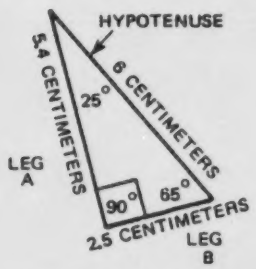


Figure 10. Right triangle.

This relationship is the well-known Pythagorean theorem: "The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse." Figure 10 contains the approximate measurements of the legs and hypotenuse. These measurements can be verified by measurement with a centimeter scale. The use of the Pythagorean theorem to solve a problem is shown in Example C.

EXAMPLE C: PYTHAGOREAN THEOREM.		
Given:	The two legs of a right triangle are the same length as those in Figure 10.	
Find:	The hypotenuse.	
Solution:	$(\text{hypotenuse})^2 = (\text{leg A})^2 + (\text{leg B})^2$	: The Pythagorean theorem
	$(\text{hypotenuse})^2 = (5.4\ \text{cm})^2 + (2.5\ \text{cm})^2$	
	$(\text{hypotenuse})^2 = 29.16\ \text{cm}^2 + 6.25\ \text{cm}^2$	
	$(\text{hypotenuse})^2 = 35.41\ \text{cm}^2$	
	Take the square root of both sides:	
	hypotenuse = 5.95 cm, or approximately 6 cm	

TRIGONOMETRIC RATIOS — SINE, COSINE, AND TANGENT

Certain ratios of lengths of sides of a right triangle are of fundamental importance in technical work. Before these important ratios can be explained, some new terminology associated with right triangles must be defined.

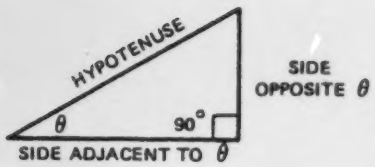


Figure 11. Trigonometric components.

Figure 11, a right triangle is shown with one of the acute angles, θ (pronounced "thay-tuh"). The legs of the right triangle now are named in terms of their location relative to a particular angle of the right triangle. One side is opposite angle θ (across the triangle from θ). The other leg is the side adjacent to angle θ (next to the angle θ). The side adjacent to θ is one of the lines that defines the angle θ. The hypotenuse, of course, is still the side across from the 90° angle. The ratios of interest are the following:

- Length of side opposite θ/length of hypotenuse.
- Length of side adjacent to θ/length of hypotenuse.
- Length of side opposite θ/length of side adjacent to θ.

These ratios are constant for a given angle θ and are given the following names and abbreviations (abbr.):

- Sine $\theta = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}}$ (abbr. sin θ)
- Cosine $\theta = \frac{\text{length of side adjacent to } \theta}{\text{length of hypotenuse}}$ (abbr. cos θ)
- Tangent $\theta = \frac{\text{length of side opposite to } \theta}{\text{length of side adjacent to } \theta}$ (abbr. tan θ)

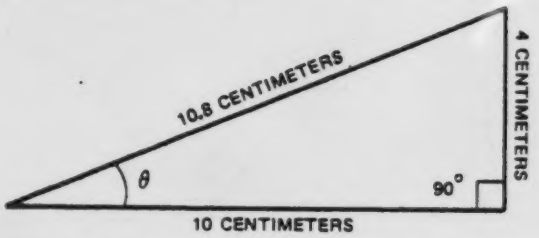
These names are read "sine of theta, cosine of theta, and tangent of theta." They should be committed to memory because of their extensive and frequent use in technology. Frequently, other Greek letters are used to label angles. These are some of the common letters used for angles:

α (alpha) – pronounced "al-fuh."
β (beta) – pronounced "bay-tuh."
γ (gamma) – pronounced "gam-muh."
φ (phi) – pronounced "fee."

Example D demonstrates how to find the sine, cosine, and tangent ratios for a given right triangle.

EXAMPLE D: SINE, COSINE, AND TANGENT RATIOS FOR A GIVEN ANGLE OF A RIGHT TRIANGLE.

Given: A right triangle has angles and sides with values shown in sketch.



Find: Sin θ, Cosine θ, Tangent θ.

Solution:

$$\text{Sin } \theta = \frac{\text{length of side opposite to } \theta}{\text{length of hypotenuse}} = \frac{4 \text{ cm}}{10.8 \text{ cm}}$$

$$\text{Sin } \theta = 0.370$$

$$\text{Cos } \theta = \frac{\text{length of side adjacent to } \theta}{\text{length of hypotenuse}} = \frac{10 \text{ cm}}{10.8 \text{ cm}}$$

$$\text{Cos } \theta = 0.925$$

$$\text{Tan } \theta = \frac{\text{length of side opposite to } \theta}{\text{length of side adjacent to } \theta} = \frac{4 \text{ cm}}{10 \text{ cm}}$$

$$\text{Tan } \theta = 0.400$$

The angle to which each ratio is related is approximately 22 degrees. This value can be verified by protractor measurements.

Note that in Example D, the values of the ratios are pure numbers and are directly related to the same angle. Any right triangle, regardless of size, containing the same 22-degree angle as in Example D would have the same values for the three trigonometric ratios of sine, cosine, and tangent.

The calculated ratios are useful if they are available for all acute angles. These ratios have been determined and arranged in tables. Table 1 illustrates the sine, cosine, and tangent ratio values for all acute angles from 0 to 90 degrees in one degree increments. The values of the sine and cosine are never greater than 1.000 or less than 0 for acute angles. The tangent value is greater than 1 for angles above 45°, and becomes extremely large for angles approaching 90°.

TABLE 1. NATURAL TRIGONOMETRIC FUNCTIONS.

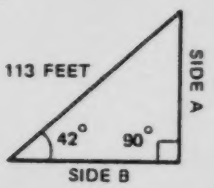
Angle		Sine	Co-sine	Tan-gent	Angle		Sine	Co-sine	Tan-gent
De-Gree	Ra-dian				De-gree	Ra-dian			
0°	.000	0.000	1.000	0.000					
1°	.017	.018	1.000	.018	46°	0.803	0.719	0.695	1.036
2°	.035	.035	0.999	.035	47°	.820	.731	.682	1.072
3°	.052	.052	.999	.052	48°	.838	.743	.669	1.111
4°	.070	.070	.998	.070	49°	.855	.755	.656	1.150
5°	.087	.087	.996	.088	50°	.873	.766	.643	1.192
6°	.105	.105	.995	.105	51°	.890	.777	.629	1.235
7°	1.22	.122	.993	.123	52°	.908	.788	.616	1.280
8°	.140	.139	.990	.141	53°	.925	.799	.602	1.327
9°	.157	.156	.988	.158	54°	.942	.809	.588	1.376
10°	.175	.174	.985	.176	55°	.960	.819	.574	1.428
11°	.192	.191	.982	.194	56°	.977	.829	.559	1.483
12°	.209	.208	.978	.213	57°	.995	.839	.545	1.540
13°	.227	.225	.974	.231	58°	1.012	.848	.530	1.600
14°	.244	.242	.970	.249	59°	1.030	.857	.515	1.664
15°	.262	.259	.966	.268	60°	1.047	.866	.500	1.732
16°	.279	.276	.961	.287	61°	1.065	.875	.485	1.804
17°	.297	.292	.956	.306	62°	1.082	.883	.470	1.881
18°	.314	.309	.951	.325	63°	1.100	.891	.454	1.963
19°	.332	.326	.946	.344	64°	1.117	.899	.438	2.050
20°	.349	.342	.940	.364	65°	1.134	.906	.423	2.145
21°	.367	.358	.934	.384	66°	1.152	.914	.407	2.246
22°	.384	.375	.927	.404	67°	1.169	.921	.391	2.356
23°	.401	.391	.921	.425	68°	1.187	.927	.375	2.475
24°	.419	.407	.914	.445	69°	1.204	.934	.358	2.605
25°	.436	.423	.906	.466	70°	1.222	.940	.342	2.747
26°	.454	.438	.899	.488	71°	1.239	.946	.326	2.904
27°	.471	.454	.891	.510	72°	1.257	.951	.309	3.078
28°	.489	.470	.883	.532	73°	1.274	.956	.292	3.271
29°	.506	.485	.875	.554	74°	1.292	.961	.276	3.487
30°	.524	.500	.866	.577	75°	1.309	.966	.259	3.732
31°	.541	.515	.837	.601	76°	1.326	.970	.242	4.011
32°	.559	.530	.848	.625	77°	1.344	.974	.225	4.331
33°	.576	.545	.839	.649	78°	1.361	.978	.208	4.705
34°	.593	.559	.829	.675	79°	1.379	.982	.191	5.145
35°	.611	.574	.819	.700	80°	1.396	.985	.174	5.671
36°	.628	.588	.809	.727	81°	1.414	.988	.156	6.314
37°	.646	.602	.799	.754	82°	1.431	.990	.139	7.115
38°	.663	.616	.788	.781	83°	1.449	.993	.122	8.144
39°	.681	.629	.777	.810	84°	1.466	.995	.105	9.514
40°	.698	.643	.766	.839	85°	1.484	.996	.087	11.43
41°	.716	.658	.755	.869	86°	1.501	.998	.070	14.30
42°	.733	.669	.743	.900	87°	1.518	.999	.052	19.08
43°	.751	.682	.731	.933	88°	1.536	.999	.035	28.64
44°	.768	.695	.719	.966	89°	1.553	1.000	.018	57.28
45°	.785	.707	.707	1.000	90°	1.571	1.000	.000	∞

Many modern-day electronic calculators have sin, cos, and tan buttons that enable the user to obtain the trigonometric ratios. Usually, the angle is entered into the calculator keyboard in degrees; then the desired trigonometric ratio button is pushed ([sin], [cos], or [tan]). After a short time, the value of the desired ratio appears on the calculator display. The owner's manual that comes with most electronic calculators is helpful in determining the right sequence of operations.

Frequently, the use of sin, cos, and tan ratios plays an important role in solving right-triangle problems such as (a) finding unknown sides of right triangles and (b) finding unknown angles of right triangles. In both types of problems, the unknown quantity must be defined in terms of the defining equations for the trig ratios. Examples E and F illustrate how an unknown side and an unknown angle of a right triangle are found.

EXAMPLE E: FINDING UNKNOWN SIDES OF A RIGHT TRIANGLE.

Given: A right triangle has the values shown in the adjoining sketch.



Find: The length of sides A and B.

Solution: Recall the definitions of sine and cosine:

$$\sin 42^\circ = \frac{\text{side A}}{113 \text{ ft}} \qquad \cos 42^\circ = \frac{\text{side B}}{113 \text{ ft}}$$

Using Table 1 or an electronic calculator, one obtains the values:

$$\sin 42^\circ = 0.669 \qquad \cos 42^\circ = 0.743$$

Therefore —

$$0.669 = \frac{\text{side A}}{113 \text{ ft}} \qquad 0.743 = \frac{\text{side B}}{113 \text{ ft}}$$

$$0.669\ (113 \text{ ft}) = \text{side A} \qquad 0.743\ (113 \text{ ft}) = \text{side B}$$

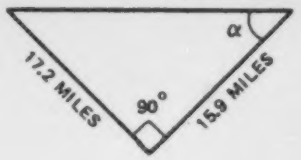
$$75.6 \text{ ft} = \text{side A} \qquad 84 \text{ ft} = \text{side B}$$

The solution can be checked independently by application of the Pythagorean theorem:

$$(75.6 \text{ ft})^2 + (84 \text{ ft})^2 = (113 \text{ ft})^2$$

EXAMPLE F: FINDING AN UNKNOWN ANGLE OF A RIGHT TRIANGLE.

Given: A right triangle has values shown in adjoining sketch.



Find: The angle α in degrees.

Solution: The side opposite angle α is known (17.2 miles), and the side adjacent to α is known (15.9 miles). These two sides define the tangent ratio of angle α; therefore —

$$\tan \alpha = \frac{17.2 \text{ miles}}{15.9 \text{ miles}}$$

$$\tan \alpha = 1.08$$

Use the tangent column of Table 1 to locate value near 1.08; then read angle:

$$\alpha = 47.2 \text{ degrees}$$

In Example F, the angle was found by use of a known tangent value. This can be accomplished approximately by use of Table 1 for sines, cosines, and tangents, or by use of an electronic calculator. Before this operation can be accomplished, the inverse trigonometric functions must be defined.

Often, as in Example F, the "inverse" operation is desirable; namely, given a trig ratio, one needs to find the associated angle. Inverse trigonometric ratio definitions frequently are written in the form shown here:

INVERSE RATIO		RATIO
$\sin^{-1}(\frac{A}{B}) = \theta$	where	$\sin \theta = \frac{A}{B}$
$\cos^{-1}(\frac{C}{B}) = \theta$	where	$\cos \theta = \frac{C}{B}$
$\tan^{-1}(\frac{A}{C}) = \theta$	where	$\tan \theta = \frac{A}{C}$

The inverse ratio is indicated by a superscript (which is not a power) on the ratio name:

$\sin^{-1}(\frac{A}{B})$ is read as "the angle whose sine is $\frac{A}{B}$"

$\cos^{-1}(\frac{C}{B})$ is read as "the angle whose cosine is $\frac{C}{B}$"

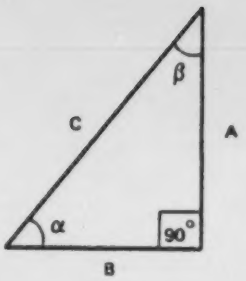
$\tan^{-1}(\frac{A}{C})$ is read as "the angle whose tangent is $\frac{A}{C}$"

Example G contains several inverse trigonometric ratios.

EXAMPLE G: INVERSE TRIGONOMETRIC RATIOS.		
Given:	a.	$\sin \alpha = 0.4226.$
	b.	$\cos \beta = 0.7771.$
	c.	$\tan \gamma = 0.3057.$
	d.	$\tan \phi = 2.475.$
Find:	a.	α
	b.	β
	c.	γ
	d.	ϕ
Solution:	a.	$\alpha = \sin^{-1}(0.4226) = 25°$
	b.	$\beta = \cos^{-1}(0.7771) = 39°$
	c.	$\gamma = \tan^{-1}(0.3057) = 17°$
	d.	$\phi = \tan^{-1}(2.475) = 68°$

Many electronic calculators provide a means of obtaining inverse trigonometric ratios. Usually the ratio is entered into the calculator, and an [INV] button is pushed before pushing the respective trig ratio button [sin], [cos], or [tan]. The owner's manual for the calculator should give the proper sequence of operations. Example G can be used for practice with inverse trig ratios.

Another helpful relationship is that for a right triangle the sine of one of the acute angles is equal to the cosine of the other. This and other relationships between the trigonometric functions for a right triangle are shown in Figure 12.



$$\sin \alpha = \frac{A}{C} \qquad \sin \beta = \frac{B}{C}$$

$$\cos \alpha = \frac{B}{C} \qquad \cos \beta = \frac{A}{C}$$

$$\tan \alpha = \frac{A}{B} \qquad \tan \beta = \frac{B}{A}$$

$$\sin \alpha = \cos \beta$$

$$\cos \alpha = \sin \beta$$

$$\tan \alpha = \frac{1}{\tan \beta}$$

Figure 12. Trigonometric functions of a right triangle.

VECTORS

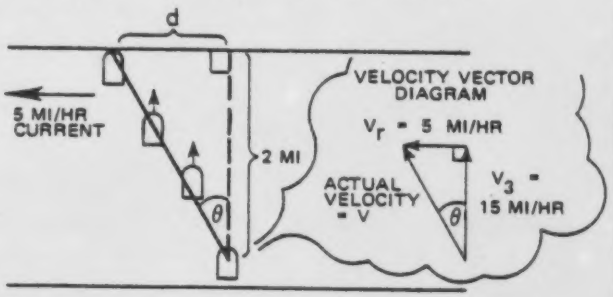
A vector is a quantity that must be specified by (1) a magnitude and (2) an angle relative to some chosen reference direction. Vectors represent many physical quantities, such as displacement, velocity, acceleration, force, and torque. Other physical quantities, such as work, energy, power, and resistance, are specified by an amount only and are referred to as "scalars."

When vector quantities are added, they yield another vector quantity that is not necessarily in the same direction as any of the original vectors. This situation is considered in Example H.

EXAMPLE H: VECTOR ADDITION BY USE OF TRIG FUNCTIONS.

Given: A boat is moving at a speed of 15 mi/hr and aimed straight across a river flowing perpendicular to the boat's intended path, at a speed of 5 mi/hr. The river is 2 miles wide.

Find:
a. Distance from the starting point to the point at which the boat will land across the river.
b. Velocity of the boat with respect to an observer on shore.



Solution: a. Notice that the distance triangle and the velocity triangle are similar right triangles. The heading angle θ from the velocity diagram is –

$$\theta = \tan^{-1}\left(\frac{5 \text{ mi/hr}}{15 \text{ mi/hr}}\right)$$

θ is the same angle in the distance triangle. In the distance triangle –

$$\theta = \tan^{-1}\left(\frac{d}{2 \text{ mi}}\right)$$

Therefore – $\left(\frac{5 \text{ mi/hr}}{15 \text{ mi/hr}}\right) = \tan\theta = \left(\frac{d}{2 \text{ mi}}\right)$

$$\left(\frac{1}{3}\right) = \left(\frac{d}{2 \text{ mi}}\right)$$

$$d = 2/3 \text{ mile}$$

b. Heading angle: $\theta = \tan^{-1}\left(\frac{5 \text{ mi/hr}}{15 \text{ mi/hr}}\right) = \tan^{-1}\left(\frac{1}{3}\right)$

$$\theta = 18.4 \text{ degrees}$$

The actual velocity of the boat can be found by use of the definitions of sin θ, cos θ, or Pythagorean theorem. Here the sin function is chosen:

$$\sin\theta = \frac{\text{velocity of river}}{\text{actual velocity}}$$

$$\sin\theta = \frac{v_r}{v_a}$$

since $\theta = 18.4$ degrees and $v_r = 5$ mi/hr

Actual velocity – $v_a = \frac{5 \text{ mi/hr}}{\sin(18.4°)}$

$$= \frac{5 \text{ mi/hr}}{0.3156}$$

$v_a = 15.84$ mi/hr in a direction 18.4 degrees downstream from straight across.

A frequent application of trigonometric ratios occurs in the process of resolving a vector into two right-angle component vectors. Example H illustrates how two right-angle vectors add to give one resulting vector. (Remember the velocity vector diagram.) Resolving a vector into its right-angle components is the inverse problem. This inverse problem can be stated simply: Given a vector and its angle relative to a reference direction, find two right-angle component vectors that would add "vectorially" to yield the given vector. One of these component vectors points in a direction parallel to the reference direction. These relationships are illustrated in Figure 13.

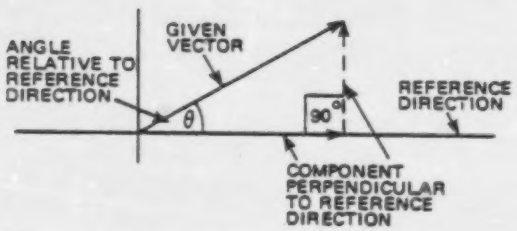


Figure 13. Resolution of a vector.

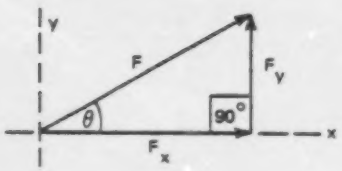


Figure 14. Vector triangle.

The right triangle formed by the vectors in Figure 13 is used as a reference to find the magnitudes of the component vectors. This vector triangle is shown symbolically in Figure 14. Application of the definition of sine θ and cosine θ to the triangle in Figure 14 results in Equations 1 and 2.

$$\sin\theta = \frac{F_y}{F} \qquad \cos\theta = \frac{F_x}{F}$$

$$\left[\begin{array}{l}\text{the magnitude}\\ \text{of component}\\ \text{opposite } \theta\end{array}\right] = \left[\begin{array}{l}\text{magnitude}\\ \text{of vector}\end{array}\right] \times \sin\theta$$

$$F_y = F \sin\theta \qquad (1)$$

and

$$\left[\begin{array}{l}\text{the magnitude}\\ \text{of component}\\ \text{adjacent to } \theta\end{array}\right] = \left[\begin{array}{l}\text{magnitude}\\ \text{of vector}\end{array}\right] \times \cos\theta$$

$$F_x = F \cos\theta \qquad (2)$$

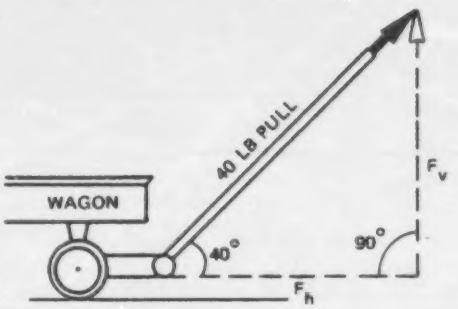
Notice the relationship between the trigonometric ratio and the component desired:

EXAMPLE I: FINDING VECTOR COMPONENTS BY USE OF TRIGONOMETRIC RATIOS.

Given: A 40-lb pull at 40° is applied along the tongue of a wagon, as illustrated below.

F_v = amount of vertical lift caused by the 40-lb pull at 40°.

F_h = amount of horizontal force tending to cause the wagon to roll.



Find: The vertical and horizontal components (F_h, F_v) of the 40-lb pull acting at 40 degrees to the horizontal direction.

Solution: Use Equations 1 and 2:

$F_v = F \sin \theta$ $\qquad$ $F_h = F \cos \theta$

$F_v = (40\ \text{lb})(\sin 40°)$ $\qquad$ $F_h = (40\ \text{lb})(\cos 40°)$

Obtain the trig ratio from Table 1 or from electronic calculator:

$F_v = (40\ \text{lb})(0.6428)$ $\qquad$ $F_h = (40\ \text{lb})(0.7660)$

$F_v = 26$ lb (rounded to nearest pound) $\qquad$ $F_h = 31$ lb (rounded to nearest pound)

These components are the combined affects on the wagon tongue caused by the single 40-lb force acting at 40 degrees. Notice that 26 lb and 31 lb do not add algebraically to give 40 lb. The components add vectorially to give a 40-lb force at 40°.

The magnitude of the resultant force is

$$F = \sqrt{F_v^2 + F_h^2}$$

$$= \sqrt{(26\ \text{lb})^2 + (31\ \text{lb})^2}$$

$$= \sqrt{1637\ \text{lb}^2}$$

F = 40 lb (rounded to the nearest pound)

SUMMARY

Trigonometry, which is concerned with the relations between the angles and sides of triangles and with functions based on these relations, is an indispensable mathematical tool in practically all branches of science and engineering. For example, many of the problems from physics and mechanics involving force, velocities, and displacements are simplified by the application of trigonometry. Trigonometric measurement is indirect; for instance, with trigonometry one can measure the height of a cliff without ever climbing it and measure the width of a river without ever crossing it. Even in business and economics, in fact in any situation where the available data seem to suggest periodic phenomena, trigonometry can be a useful aid. Vectors are an extremely important tool in technology.

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McHale, Thomas J. and Witzke, Paul T. Basic Trigonometry. Milwaukee Area Technical College Mathematics Series, No. 3. Reading, MA: Addison-Wesley Publishing Company, 1971.

Palmer, Claude Irwin and Bibb, Samuel F. Practical Mathematics. 5th ed. New York: McGraw-Hill Book Company, 1970.

Paul, Richard S. and Shaevel, Leonard M. Essentials of Technical Mathematics. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1974.

MATERIALS

Protractor (0-180 x 1 degree)
Metric scale, flat plastic (1 ft length, 0.1 cm)
Electronic calculator with trig functions (optional)

1. Using a protractor, measure θ, β, and γ; record below.

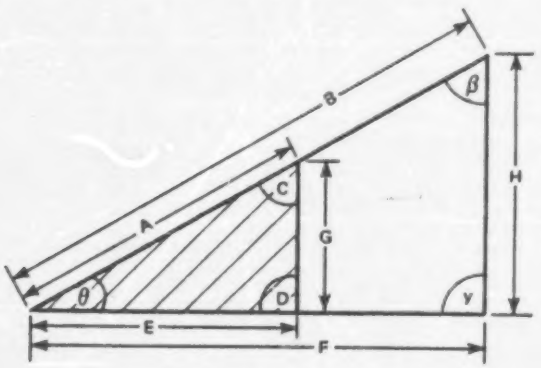


Figure 15.

   a. θ = _____ degrees.
      β = _____ degrees.
      γ = _____ degrees.

2. Use a protractor to measure angle C and D; record below.
   a. C = _____ degrees.
   b. D = _____ degrees.

3. Use a metric scale to measure the following lengths labeled in the above figure, and record them.
   a. A = _____ cm
   b. G = _____ cm
   c. E = _____ cm
   d. B = _____ cm
   e. H = _____ cm
   f. F = _____ cm

4. Calculate the following ratios, using the measurements in Step 3:

   a. $\sin\theta = \frac{G}{A} = \frac{(\quad\quad)}{(\quad\quad)} =$ _____.

   b. $\sin\theta = \frac{H}{B} = \frac{(\quad\quad)}{(\quad\quad)} =$ _____.

   c. $\cos\theta = \frac{E}{A} = \frac{(\quad\quad)}{(\quad\quad)} =$ _____.

   d. $\cos\theta = \frac{F}{B} = \frac{(\quad\quad)}{(\quad\quad)} =$ _____.

   e. $\tan\theta = \frac{G}{E} = \frac{(\quad\quad)}{(\quad\quad)} =$ _____

   f. $\tan\theta = \frac{H}{F} = \frac{(\quad\quad)}{(\quad\quad)} =$ _____

5. Use Table 1 or an electronic calculator to find the angle to the nearest degree for each of trig ratios in Step 4.

   a. $\theta =$ _____ degrees.

   b. $\theta =$ _____ degrees.

   c. $\theta =$ _____ degrees.

   d. $\theta =$ _____ degrees.

   e. $\theta =$ _____ degrees.

   f. $\theta =$ _____ degrees.

6. From the data gathered in Problems 1-5, verify these facts for similar triangles:

   a. The ratio of corresponding sides of similar triangles are equal.

   $\frac{B}{A} =$ _____, $\frac{H}{G} =$ _____, $\frac{F}{E} =$ _____.

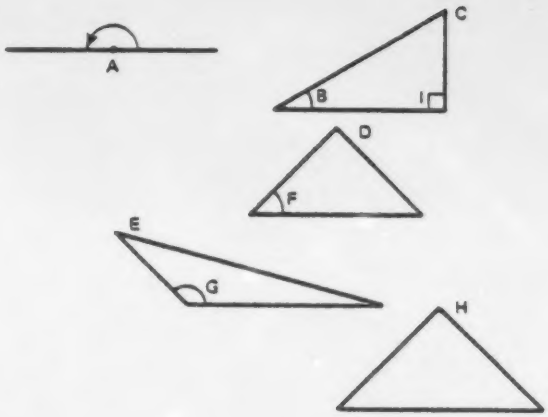
   b. All angles of a triangle add to equal 180°.

   $\angle\theta + \angle C + \angle D = ?$

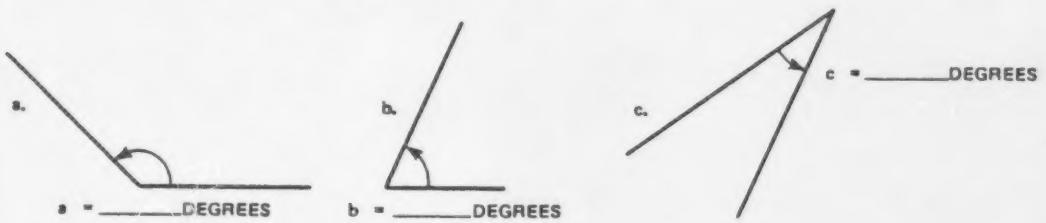
   $\angle\theta + \angle B + \angle r = ?$

7. Identify the following terms from the adjoining sketches:

   a. An angle: B example).
   b. Right angle: ____.
   c. Straight angle: ____.
   d. Acute angle: ____.
   e. Obtuse angle: ____.
   f. A triangle: ____.
   g. Right triangle: ____.
   h. Acute triangle: ____.
   i. Obtuse triangle: ____.
   j. Similar triangles: ____.
      and ____.



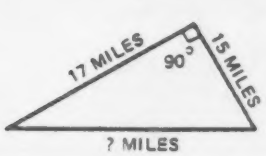
8. Use a protractor to measure the following angles. Record degree of each under corresponding sketch.



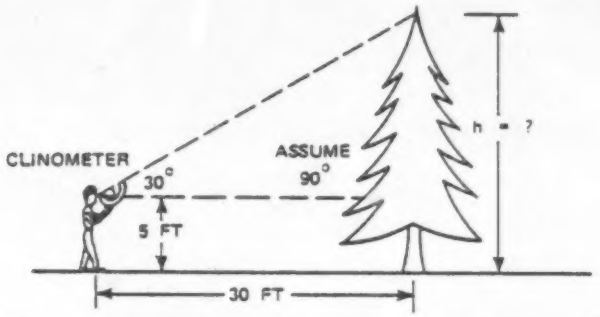
9. Define the following terms, and use diagrams to illustrate definitions if needed:

   a. A degree.
   b. The sine of an angle.
   c. The cosine of an angle.
   d. The tangent of an angle.

10. For the right triangle below, calculate the hypotenuse with the Pythagorean theorem:

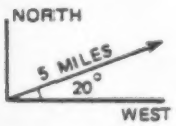


11. Bob Crachett wants to know how tall the Christmas tree is before he chops it down to take home. He measures the angle with a clinometer as shown in the diagram, to be 30° while he stands 30 ft from the tree. From the ground to Bob's eye level is 5 ft. Calculate the height (h) of the tree for Bob.



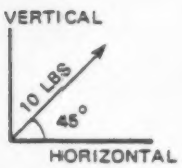
12. Resolve the following vectors into components along the indicated directions:

a.



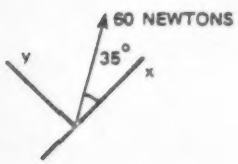
(along north and west)

b.



(along vertical and horizontal)

c.



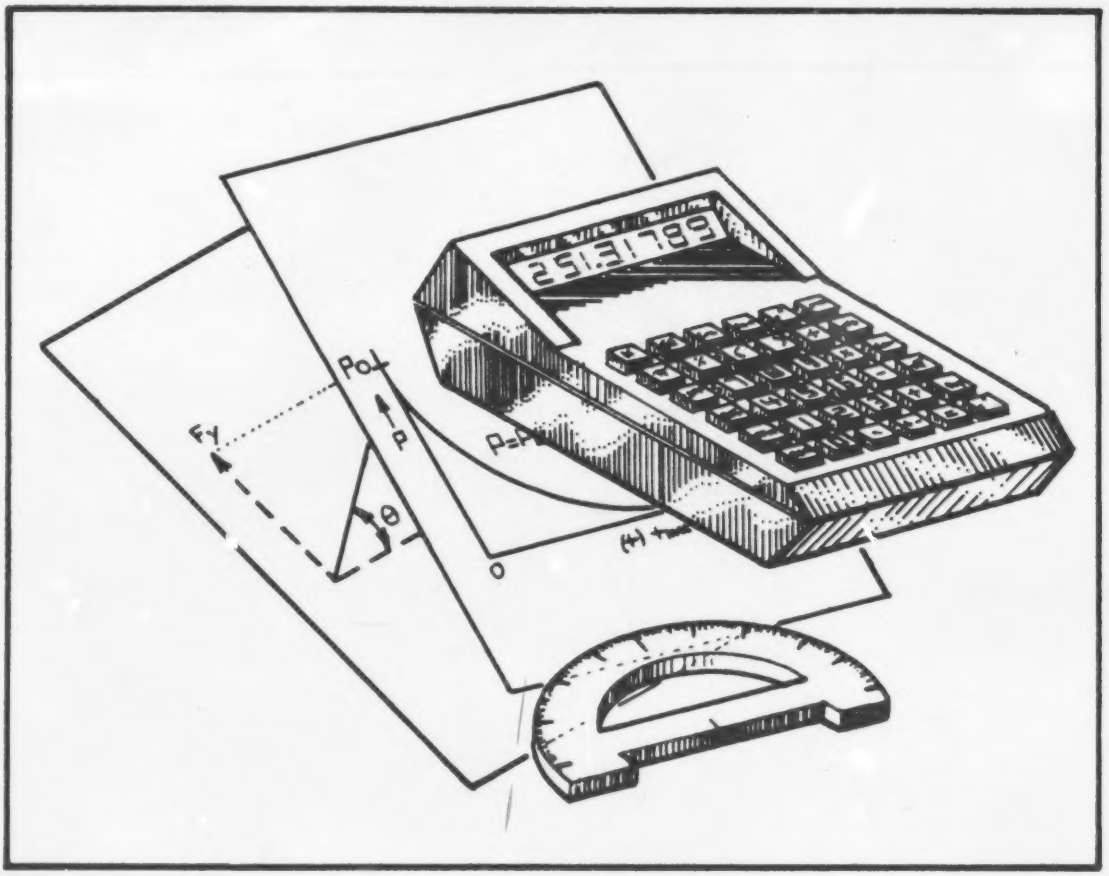
(along x and y)

UTC

UNIFIED TECHNICAL CONCEPTS

MODULE S 5

VECTORS AND SCALARS



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INTRODUCTION

Basic physical quantities such as speed, temperature, mass, and pressure require only a magnitude to specify their values. They are called "scalar quantities," or simply "scalars." Other basic physical quantities such as force, velocity, acceleration, and torque require both a magnitude and a direction to specify their values. They are called "vector quantities," or simply "vectors."

Scalars and vectors play important roles in science and engineering. To know the difference between them and to be able to describe them correctly when using them is essential. This module defines scalars and vectors and distinguishes carefully between them. Different methods of combining (adding or subtracting) scalars and vectors are outlined in detail.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. List at least four common examples of the following:
   a. Scalar quantities.
   b. Vector quantities.
2. Given a vector and a scalar quantity, graphically show the results when the vector is multiplied or divided by the scalar.
3. Given two or more vector quantities, add and subtract the vectors graphically, with the aid of a scale and protractor, to obtain a resultant vector. Solve the same mathematically using trigonometry.
4. Given a vector quantity with magnitude and direction specified, resolve the vector graphically, with the aid of a scale and protractor, into two components at right angles to each other. Solve the same problem mathematically using trigonometry.

"Scalar quantities" are defined as "quantities completely specified only by their magnitude (amount)." The number and its unit are all that is needed to represent a scalar quantity. Some examples of scalar quantities are mass (12 g), area (5 sq mi), volume (144 cu in), density (2.7 grams per cubic cm), distance (12 mi), and speed (30 mi/hr). Any quantity represented as a count of objects, such as a dozen eggs in a box, money in a pocket, and a number of birds in a flock, is a scalar quantity. We may recall encountering such problems as those illustrated in Figure 1 while studying arithmetic in Elementary School. These problems are simple examples of scalar addition, in which the magnitudes (sizes) of the quantities are added together. In scalar subtraction, the magnitudes are subtracted from each other.

30 CENTS + 40 CENTS = 70 CENTS

40 APPLES − 30 APPLES = 10 APPLES

Figure 1. Addition of scalar quantities.

In engineering and technical work, a different type of quantity often is used in problem solving. This quantity, called a "vector," has its own rules for addition and subtraction.

VECTORS

A "vector quantity" is one that completely is specified by a magnitude and a direction. It consists, therefore, of a number, a unit, and a direction. Some examples of vector quantities are displacement (20 mi, North), velocity (180 mi/hr, 30° South of West), and force (100 lb, 30° above horizontal). Other examples of vector quantities are acceleration, momentum, torque, and electric fields. Vectors contain scalar magnitudes in their specifications. Several vector quantities often are confused with similar scalar quantities; however, the names of the quantities should distinguish between them. Two examples of scalar and vector terms often confused are shown in Table 1.

TABLE 1. SCALAR AND VECTOR QUANTITIES.

Name of Scalar Quantity	Name of Vector Quantity
Distance (30 miles)	Displacement (30 miles, West)
Speed or Rate (60 mi/hr)	Velocity (60 mi/hr, 20° West of North)

The direction of the vector is an important aspect and always should be included when specifying the vector. The direction of a vector usually is given relative to a reference direction, such as X-axis or an eastward line. Figure 2 consists of a vector diagram that represents a displacement. The angle 45° specifies the vector's direction, or heading, in the right-angle coordinate system of NORTH, EAST, WEST, SOUTH. Another coordinate system frequently used is the common X-Y axes at 90° to one another.

In a scale drawing of a vector quantity, the length of the line represents the magnitude of the vector quantity. The orientation of the line on the page represents its "line of direction." The arrow-tip drawn at one end of the vector indicates the "sense" of the vector along the line of direction. These characteristics are illustrated in Figure 3.

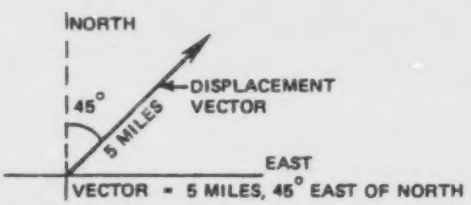


Figure 2. Displacement vector.

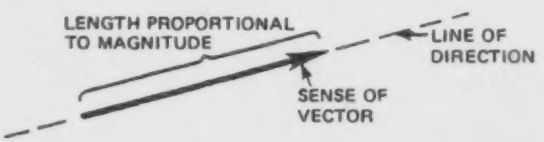


Figure 3. Vector characteristics.

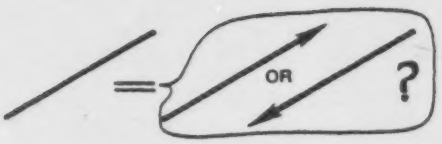


Figure 4. Incomplete vector symbol.

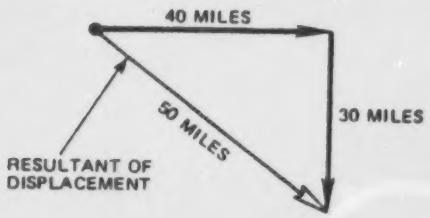


Figure 5. Vector addition.

Sometimes the arrow-tip is omitted by mistake from the end of the vector. When this error occurs, confusion arises concerning which way the vector points along the line of direction (Figure 4).

In engineering and technical work, vector quantities are added by a process called "vector addition." When vector addition is performed, both the magnitude and the direction of the quantities must be considered. In Figure 5, the vector sum is —

40 miles + 30 miles = 50 miles

The arrow is placed above the quantity to indicate that it is a vector quantity. It is also possible to combine or add a 40 mi and a 30 mi displacement to obtain a different resultant. Other possible combinations are —

vector sum: $\overrightarrow{30 \text{ miles}} + \overrightarrow{40 \text{ miles}} = \overrightarrow{70 \text{ miles}}$,

as shown in Figure 6,

and vector sum: $\overrightarrow{40 \text{ miles}} + \overrightarrow{30 \text{ miles}} = \overrightarrow{10 \text{ miles}}$,

as shown in Figure 7.

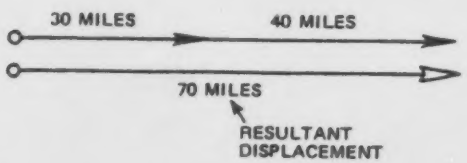


Figure 6. Linear vector addition.

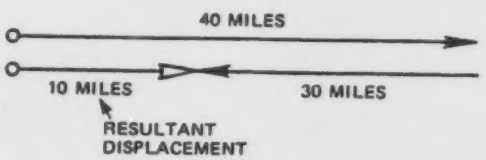


Figure 7. Vector addition.

A fourth possibility is shown in Figure 8.

vector sum: 40 miles + 30 miles = 40 miles.

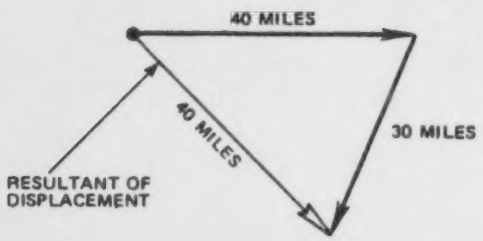


Figure 8. Vector addition.

When a 40-mi displacement and a 30-mi displacement are added, the magnitude of the resultant may vary from a minimum of 10 mi to a maximum of 70 mi, as demonstrated in Figures 6 and 7. The direction must be taken into account when displacement vectors (or any set of vectors) are combined by vector addition. Vectors can be added together to find a resultant vector — a single vector that produces the same effect as the original vectors added together.

The subtraction of vectors is also simple once the addition of vectors is understood. Figure 8 illustrated the vector sum:

$$\overrightarrow{40 \text{ miles}} + \overrightarrow{30 \text{ miles}} = \overrightarrow{40 \text{ miles}}.$$

If, on the other hand, the second vector is subtracted from the first, the equation becomes:

$$\overrightarrow{40 \text{ miles}} - \overrightarrow{30 \text{ miles}} = \overrightarrow{\text{Resultant}}$$

This same example can be treated as a vector addition problem by simply writing the equation as:

$$\overrightarrow{40 \text{ miles}} + \overrightarrow{(-30 \text{ miles})} = \overrightarrow{\text{Resultant}}$$

The vector (-30 miles) is the reverse of the vector 30 miles. It has the same magnitude and the opposite direction. Thus, the correct subtraction process would appear as shown in Figure 9.

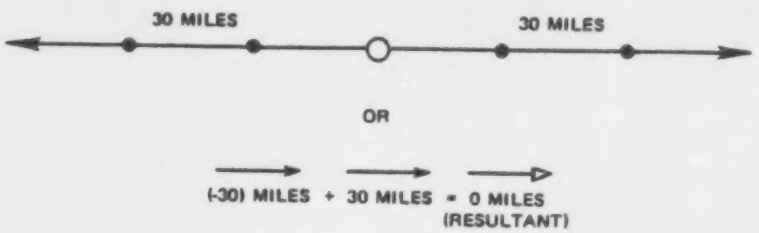


Figure 9. Vector addition.

When $\overrightarrow{40 \text{ miles}} + \overrightarrow{30 \text{ miles}}$ are added as shown in Figure 8, the resultant is 40 miles in the direction down and to the right. On the other hand, when the same two vectors are subtracted, the resultant is about 60 miles up and to the right as shown in Figure 10.

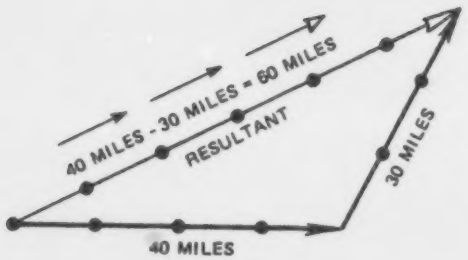


Figure 10. Vector subtraction.

Addition of a vector and a scalar, or subtraction of one from the other would be meaningless; however, vector quantities can be multiplied or divided by scalars in a particular way. The process of multiplying a vector quantity by a scalar quantity yields a resulting vector in the same direction with a different magnitude than the original vector. This process is illustrated in Figure 11. Notice that the new magnitude is merely the original magnitude multiplied by the scalar.

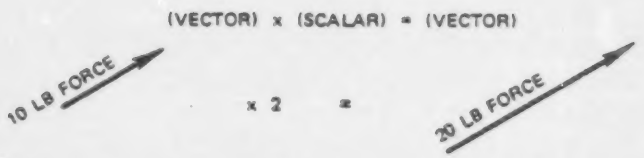


Figure 11. Multiplying a vector times a scalar.

In order to draw on paper a vector that represents a physical quantity expressed in some other units (miles, pounds of force, miles per hour), one must choose a scale with which to represent the vector as a line of some convenient length on the page. The scale establishes a simple ratio of length units to the units of the vector quantity, as illustrated in Figure 12.

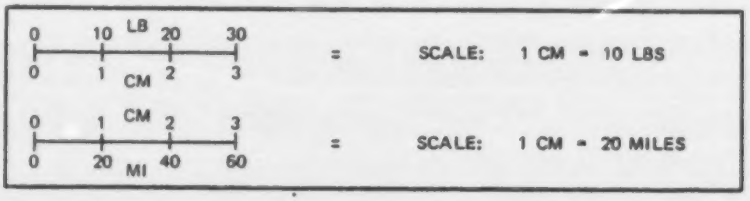


Figure 12. Scales used to relate a vector quantity to the length of its line.

A vector quantity can be converted to a line vector by simple multiplication of the vector by the appropriate scale ratio. This process is shown in Example A.

EXAMPLE A: CONVERSION OF A VECTOR QUANTITY TO A GRAPHIC VECTOR.	
Given:	A vector has a displacement of 100 mi EAST and a scale of 1 cm = 20 mi.
Find:	Length of the graphic vector in cm.
Solution:	Multiply the magnitude of the vector by the scale ratio:

$$(100 \cancel{\text{mi}}) \frac{1 \text{ cm}}{20 \cancel{\text{mi}}} = 5 \text{ cm}$$

Therefore, in a graphical vector problem using the given scale, a 100-mi vector pointing EAST would be drawn on the page as a graphic vector 5 cm in length, pointing EAST.

ADDING VECTORS BY THE GRAPHIC (POLYGON) METHOD

In adding vectors by the graphical method, the magnitude and direction of each vector is indicated by an arrow-tipped line drawn to some specified scale. In using the graphical polygon method, a ruler is employed to measure lengths, and a protractor to measure angles, as shown in Figure 13. The step-by-step procedure for adding vectors by the polygon method is as follows:

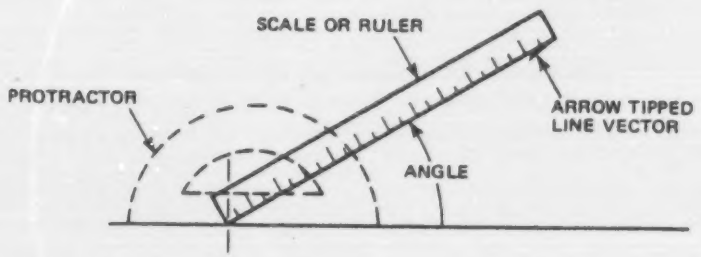


Figure 13. Length and direction of graphic vector.

1. Choose a suitable scale, and determine the length of the arrow which will represent the vector according to the specified scale.
2. Choose an origin or starting point on the paper, and draw the arrow representing the first vector. This arrow must be of proper length and must point in the appropriate direction indicated by its angle. The "tail" of the arrow always is positioned at the origin, or start-point.
3. The arrow of the second vector is drawn so that its tail is joined to the head of the first vector. The arrow of the second vector also must be drawn to scale and must point in the correct direction indicated by its angle.
4. If there is a third vector to be added, its tail is joined to the head of the second vector and correctly oriented. This process of joining tail to head is continued until the magnitude and direction of all vectors have been represented.

5. The arrow representing the resultant vector is drawn with its tail at the origin (starting point) and its head joined to the head of the last vector.
6. The resultant arrow indicates the magnitude and direction of the resultant. Measurement of the resultant arrow with the ruler and the protractor gives the magnitude and direction of the resultant vector.
7. The length of the resultant vector is converted back to the units of the original vectors by multiplying by the appropriate scale factor.

If only two vectors are added by this method, as in Figure 8, the resultant vector forms the third side of a triangle (a three-sided polygon). The polygon method can be extended to find the resultant of the addition of any number of vectors. Example B illustrates the application of this step-by-step procedure to a vector addition problem.

EXAMPLE B: ADDING VECTORS BY GRAPHICAL METHOD.

Given: A ship travels 100 mi due North on the first day of a voyage. It then travels 80 mi Northeast on the second day and 130 mi due East on the third day.

Find: The resultant displacement after three days.

Solution:

1. Establish a suitable scale of 25 mi = 1 cm.
2. Convert all vector magnitudes in the problem to graphical line lengths:

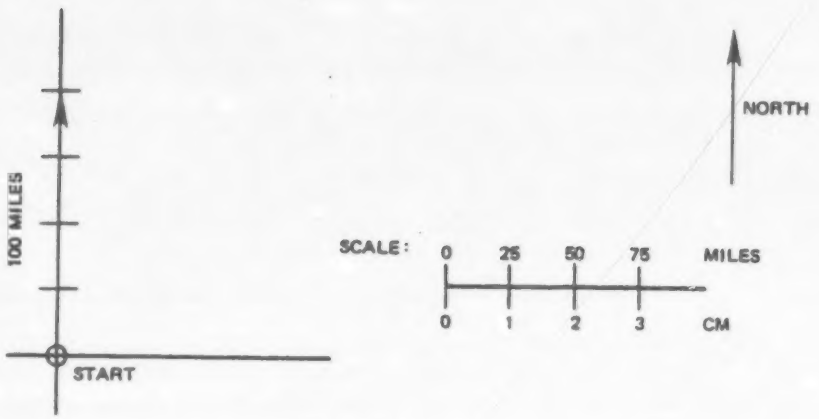
$$100 \text{ mi} \times \left(\frac{1 \text{ cm}}{25 \text{ mi}}\right) = 4 \text{ cm (North)}$$

$$80 \text{ mi} \times \left(\frac{1 \text{ cm}}{25 \text{ mi}}\right) = 3.2 \text{ cm (Northeast)}$$

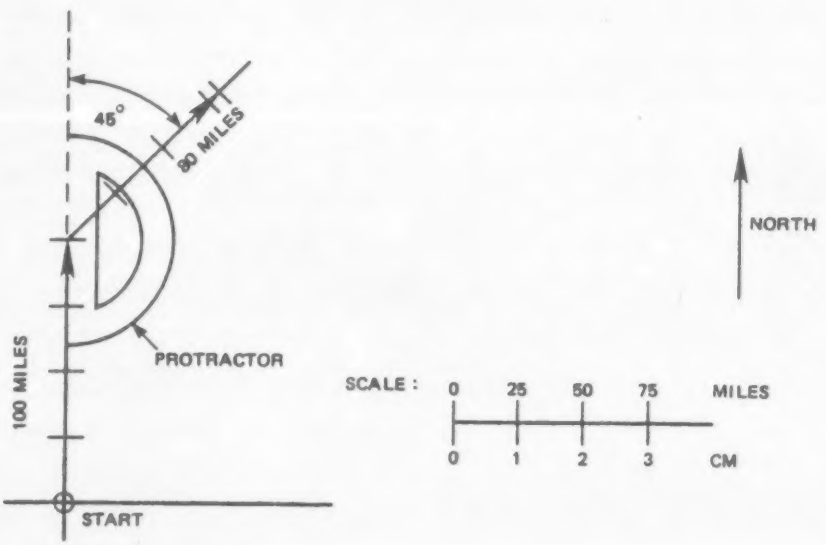
$$130 \text{ mi} \times \left(\frac{1 \text{ cm}}{25 \text{ mi}}\right) = 5.2 \text{ cm (East)}$$

NOTE: Because of reductions in copy size caused by printing methods used in reproducing these course materials, the scale of the drawings in this module may vary slightly. To ensure consistency in the measurement of vector quantities in this module which have been drawn to scale, it is essential that the ruler depicted on page 24 of this module be used as the measuring standard.

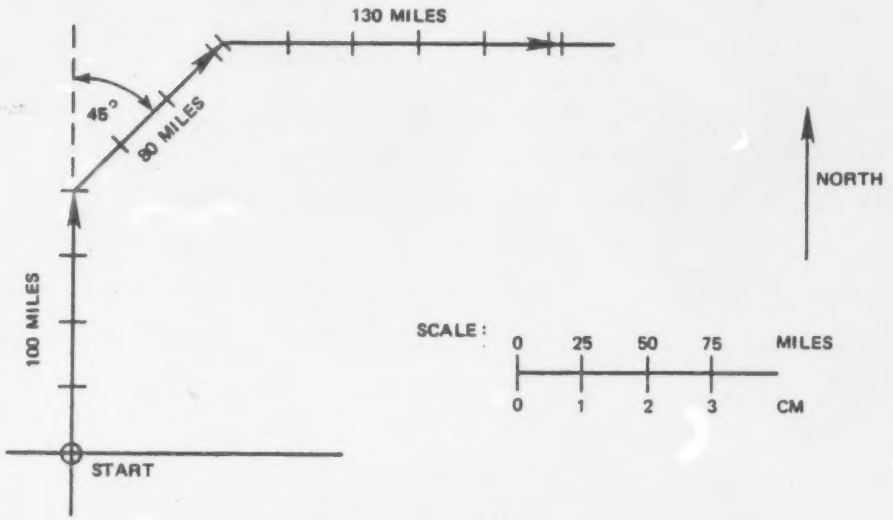
3. Choose a starting point, and draw the first 4-cm (100 mi) vector in its direction (North), as indicated by the arrow:



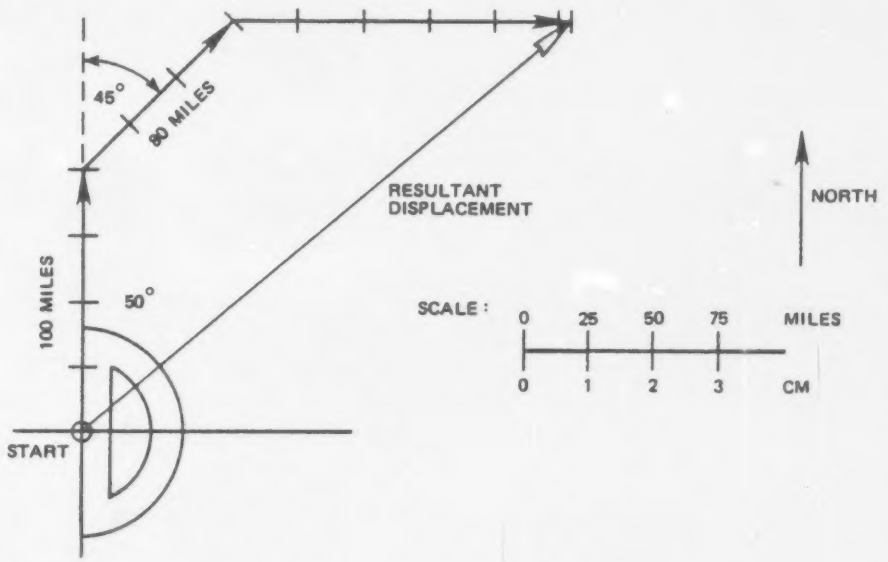
4. Measure the angle Northeast (45° in direction East of North), and draw the second 80-mi displacement (3.2 cm) as shown:



5. Draw the 130-mi vector (5.2 cm) in a due East direction by attaching its tail to the head of the 80-mi vector as shown:



6. Draw the resultant vector from the start (origin) to the tip of the 130-mi (5.2 cm) vector:



7. Measured with a ruler, the resultant vector is 9.84 cm in length; therefore, the magnitude of the resultant displacement is — $9.84 \text{ cm} \times \left(\frac{25 \text{ mi}}{1 \text{ cm}}\right) = 246 \text{ mi.}$

The direction is found with a protractor, and the angle is measured relative to North and found to be 50°. The resultant displacement, therefore is –

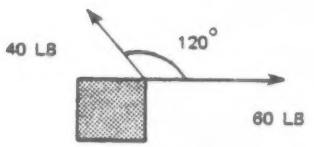
246 mi in a direction 50° East of North.

Steps 1, 2, 3, 4, and 5 of this solution illustrated the building of the graphical solution. The graphical solution actually would be done on one diagram and appear as in Step 6. Notice that on any graphical solution, the scale used must be shown near the vector polygon diagram.

The graphical polygon method can be used to find the resultant of all kinds of vectors. In Example C, this method is used to find the resultant of two force vectors.

EXAMPLE C: GRAPHICAL ADDITION OF FORCE VECTORS.

Given: A 60-lb pull and a 40-lb pull are applied to an object. The angle between these two forces is 120°.



Find: The resultant force.

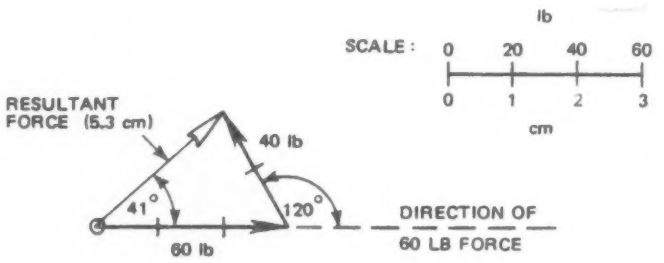
Solution:

1. Establish a scale 1 cm = 20 lb:

$$60 \cancel{\text{lb}} \times \left(\frac{1 \text{ cm}}{20 \cancel{\text{lb}}}\right) = 3 \text{ cm}$$

$$40 \cancel{\text{lb}} \times \left(\frac{1 \text{ cm}}{20 \cancel{\text{lb}}}\right) = 2 \text{ cm}$$

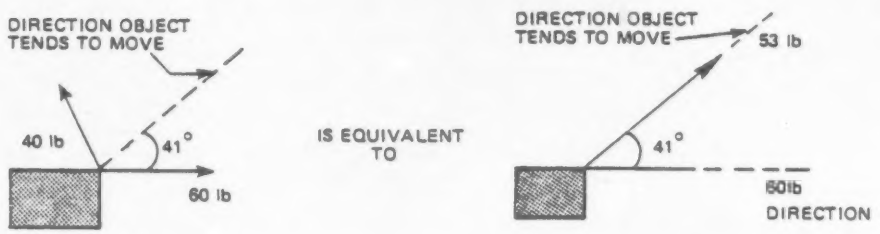
2. Addition of these two vectors by the graphical method produces the following diagram:



3. The resultant force vector = 5.3 cm; therefore,

   $2.65 \text{ cm} \times \left(\frac{20 \text{ lb}}{1 \text{ cm}}\right) = 53 \text{ lb}$

4. Using a protractor, the angle that the resultant vector makes with the direction of the 60-lb force is 41°.
5. The resultant force is 53 lb at an angle of 41° with the 60-lb force direction.
6. This means that the net effect of the 53-lb force at the angle of 41° would be the same on the object as the combined effect of the two given forces:



GRAPHICAL COMPONENTS

Often, the replacement of one vector with an equivalent set of two vectors is advantageous; this process is the reverse of the situation shown in Step 6 of Example B. The two replacement vectors, called "components," most often are desired at right angles to each other. Perpendicular components are sometimes referred to as "orthogonal components." Two such components of a vector are shown in Figure 14.

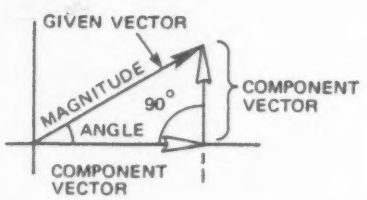


Figure 14. Vector components.

The two components add vectorially to produce the given vector. The directions of components usually are chosen to be parallel to the coordinate system directions, such as NORTH, EAST, SOUTH, WEST, and HORIZONTAL-VERTICAL. The given vector then is said to be "resolved" into the two desired components.

To find graphically the magnitude of the components of a given vector, the vector first must be scaled as in the polygon method and oriented in the proper direction relative to one of the chosen coordinate system directions. Examples of vectors whose components might be desired are shown in Figure 15 a, b, and c.

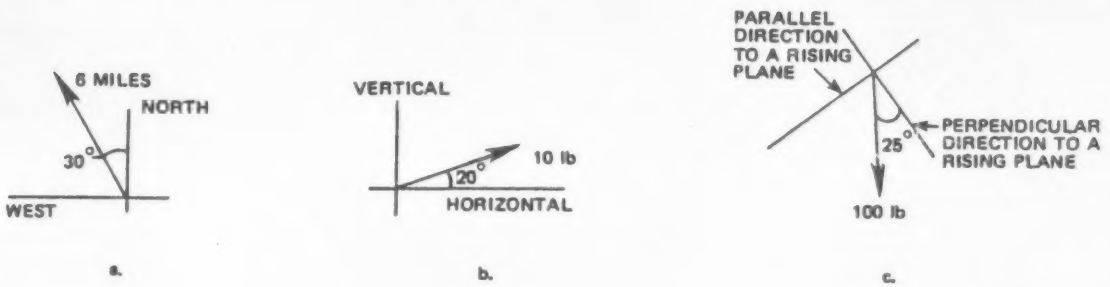
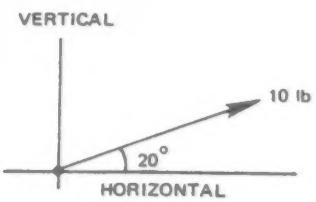


Figure 15. Resolution of vectors into orthogonal components.

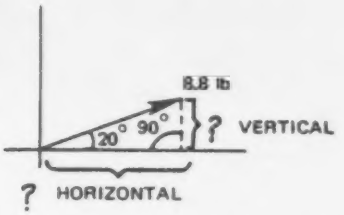
Second, a line must be drawn from the tip of the given vector perpendicular to either of the two coordinate axes. The sides of the right triangle produced can be measured with a ruler. The lengths of the two sides, converted according to the proper scale, represent the two components. Example D illustrates the technique.

EXAMPLE D: GRAPHICAL RESOLUTION OF A GIVEN VECTOR INTO ITS PERPENDICULAR COMPONENTS.

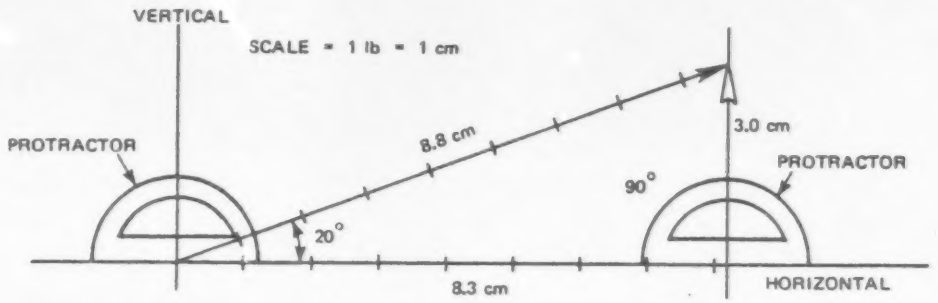
Given: An 8.8-lb force vector acts 20° above the horizontal to the right.



Find: The magnitude of its components along the horizontal and vertical directions.



Solution: 1. Scale the 8.8 lb force using 1 cm = 1 lb on a right-angle coordinate system drawn on the page at 20° as shown below:



2. Use a protractor to construct a line from the tip of the scaled vector to meet the horizontal direction at 90°.
3. Measure the sides of the right triangle formed:

   HORIZONTAL COMPONENT: 8.3 cm
   VERTICAL COMPONENT: 3.0 cm

4. Convert to original units by the scale factor:

   $8.3 \text{ cm} \times \left(\frac{1 \text{ lb}}{1 \text{ cm}}\right) = 8.3 \text{ lb}$

   $3.0 \text{ cm} \times \left(\frac{1 \text{ lb}}{1 \text{ cm}}\right) = 3.0 \text{ lb}$

5. The components have these magnitudes:

   HORIZONTAL: 8.3 lb
   VERTICAL: 3.0 lb

In Example D, an 8.3-lb force acting horizontally on an object and a 3.0-lb force acting vertically on the same object have the same effect as an 8.8-lb force acting on the object at 20° above the horizontal. Often a physical meaning can be given to these components. For example, if the 8.3-lb force of Example D were applied to a child's wagon as illustrated in Figure 16, the horizontal components of 8.3 lb may be considered to be the propelling effect of the 8.8-lb force that tends to make the wagon roll. The vertical component of 3.0 lb may be considered to be the lifting effect of the 8.8-lb force applied at the base of the tongue of the wagon. This lift would tend to make front wheels lift up as the wagon was pulled.

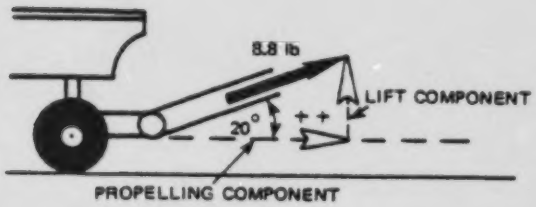


Figure 16. Pulling and lifting vector components.

MATHEMATICAL SOLUTIONS

Vector problems may be solved mathematically by employing the trigonometry functions discussed in Module S-4, "Angles and Triangles." The direction of horizontal to the right is customarily assigned an angle of 0°; all angles are measured counterclockwise from that line. (Thus, vertically upward is 90°.) The magnitude of the vertical and horizontal components of any vector of magnitude F is given by the following equations:

$$\text{Vertical Component:} \quad F_y = F \sin \theta \qquad (1)$$

$$\text{Horizontal Component:} \quad F_x = F \cos \theta \qquad (2)$$

Any number of vectors may be added mathematically by resolving them into their horizontal and vertical components, and then adding the horizontal components of all the vectors to determine the horizontal component of the resultant, and adding the vertical components to obtain the vertical component of the resultant. The magnitude and direction of the resultant vector may be found using the following equations:

$$\text{Magnitude:} \quad R = \sqrt{R_x^2 + R_y^2}$$

$$\text{Direction:} \quad \theta = \cos^{-1}\left(\frac{R_x}{R}\right) \tag{3}$$

or

$$\theta = \sin^{-1}\left(\frac{R_y}{R}\right) \tag{4}$$

Example E illustrates the use of this method to solve the same problem solved graphically in Example B.

EXAMPLE E: ADDING VECTORS BY THE MATHEMATICAL METHOD.

Given: A ship travels 100 miles due North on the first day of a voyage. It then traveled 80 miles Northeast on the second day and 130 miles due East on the third day.

Find: The resultant displacement after three days.

Solution: The three vectors are:

A = 100 mi at 90°
B = 80 mi at 45°
C = 130 mi at 0°

1. Determine the horizontal and vertical components of each vector:

$A_x = A \cos \theta$	$A_y = A \sin \theta$
$A_x = (100 \text{ mi}) \cos 90°$	$A_y = (100 \text{ mi}) \sin 90°$
$= (100 \text{ mi}) (0)$	$= (100 \text{ mi} (1)$
$A_x = 0 \text{ mi}$	$A_y = 100 \text{ mi}$
$B_x = (80 \text{ mi}) \cos 45°$	$B_y = (80 \text{ mi}) \sin 45°$
$= (80 \text{ mi}) (0.707)$	$= (80 \text{ mi}) (0.707)$
$B_x = 56.6 \text{ mi}$	$B_y = 56.6 \text{ mi}$
$C_x = (120 \text{ mi}) \cos 0°$	$C_y = (130 \text{ mi}) \sin 0°$
$= (130 \text{ mi}) (1)$	$= (130 \text{ mi}) (0)$
$C_x = 130 \text{ mi}$	$C_y = 0 \text{ mi}$

2. Add horizontal and vertical components to determine components of the resultant vector:

$$R_x = A_x + B_x + C_x$$
$$= (0) + (56.6\ \text{mi}) + (130\ \text{mi})$$
$$R_x = 186.6\ \text{mi}$$

$$R_y = A_y + B_y + C_y$$
$$= (100\ \text{mi}) + (56.6\ \text{mi}) + (0)$$
$$R_y = 156.6\ \text{mi}$$

3. Determine the magnitude of the resultant.

$$R = \sqrt{R_x^2 + R_y^2}$$
$$= \sqrt{(186.6\ \text{mi})^2 + (156.6\ \text{mi})^2}$$
$$= \sqrt{34820\ \text{mi}^2 + 24524\ \text{mi}^2}$$
$$= \sqrt{59344\ \text{mi}^2}$$
$$R = 243.6\ \text{mi}$$

4. Determine the direction of the resultant:

$$\theta = \cos^{-1}\left(\frac{R_x}{R}\right)$$
$$= \cos^{-1}\left(\frac{186.6\ \text{mi}}{243.6\ \text{mi}}\right)$$
$$= \cos^{-1}(0.7660)$$
$$\theta = 40.0°$$

The resultant displacement is 243.6 miles in a direction 40.0° North of East. This answer is more accurate than that arrived at through graphical techniques in Example B.

SUMMARY

A scalar quantity can be specified only by magnitude (size); whereas, a vector quantity is specified by both magnitude and direction. Some examples of quantities often represented as vectors are velocity, force, and acceleration. Examples of scalar quantities are time, temperature, and speed.

Vectors are represented by arrows, the length of which represents magnitude of the vector quantity. The arrowhead indicates the direction of the vector quantity. Vectors can be added to produce the "resultant;" such added vectors are called "component vectors."

Vectors and scalars find wide application in technology; for example, in physics, electronics, navigation, etc.

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Beiser, Arthur. Modern Technical Physics. 3rd ed. Menlo Park, CA: The Benjamin/Cummings Publishing Company, Inc., 1979.

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Harris, Norman C. and Hemmerling, Edwin M. Introductory Applied Physics. 3rd ed. New York: McGraw-Hill Book Company, Inc., 1972.

Hewitt, Paul G. Conceptual Physics: A New Introduction to Your Environment. 3rd ed. Boston: Little, Brown and Company, 1977. See especially Appendix III, "Vectors."

Paul, Richard S. and Shaevel, Leonard M. Essentials of Technical Mathematics. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1974.

REVIEW QUESTIONS

MATERIALS

Several sharp pencils
Notebook paper or graph paper (5 sqs to the inch)
A thin, flat plastic ruler (1 ft length, English-Metric Scale with metric scale to 1 mm)
A see-thru plastic protractor (0-180 x 1°) similar to STAEDTLER 568 50-10

Solve the following problems by the graphical methods illustrated in this module. Draw a sketch of the physical situation, and illustrate the vectors involved in each problem. Sketch simple arrow-tipped lines.

1. A 100-lb push toward the North and a 200-lb push toward the East act upon an object. Determine the magnitude and direction of the resultant force by the graphical method.
2. The following three forces are acting upon an object:
   a. 1000 lb, horizontally to the right.
   b. 1500 lb, downward and to the left at an angle of 30° below the horizontal.
   c. 500 lb, upward and to the right at an angle of 53° above the horizontal.

   Find the resultant by the polygon method.
3. A ship that can travel 30 mi/hr in still water is headed due West. A 10 mi/hr current is flowing Northeast. How fast and in what direction will the ship travel?
4. a. Resolve a 200-mi, 30° East of North displacement vector into its Easterly and Northerly components.
   b. Resolve a 100-mi. due West displacement vector into its Northerly and Westerly components.
   c. Resolve a 80-mi, 60° West of South displacement vector into its Southerly and Westerly components.
   d. Add the three displacement vectors in parts a, b, and c above by the graphical polygon method, and determine the magnitude and direction of the resultant vector.

e. Find the components of the resultant vector along the North-South line and along the East-West line.

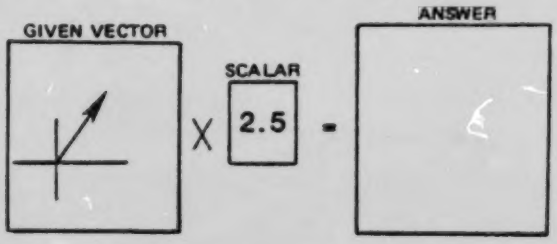
f. Show the scalar addition of the North-South components of a, b, and c that would result in the East-West component of the resultant found in e. How do they compare?

5. Discuss how vectors could be applied to several physical quantities in technology.

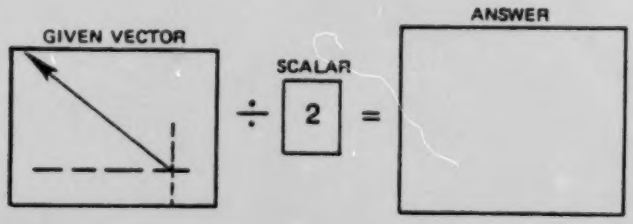
6. Define a scalar quantity, and list at least four examples of scalar quantities.

7. Define a vector quantity, and list at least four examples of vector quantities.

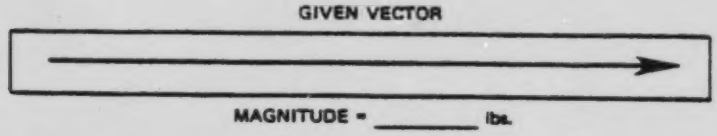
8. In the space provided below, show the result of multiplying the given vector by the scalar quantity 2.5. Use a protractor and a scale.



9. In the space provided below, graphically draw the result of dividing the given vector by the scalar quantity 2. Use a protractor and a scalar.



10. The vector drawn below represents a force vector drawn to a scale of 40 lbs = 1 cm. Measure the length of the vector with a metric scale, and state the magnitude of the force vector in pounds.



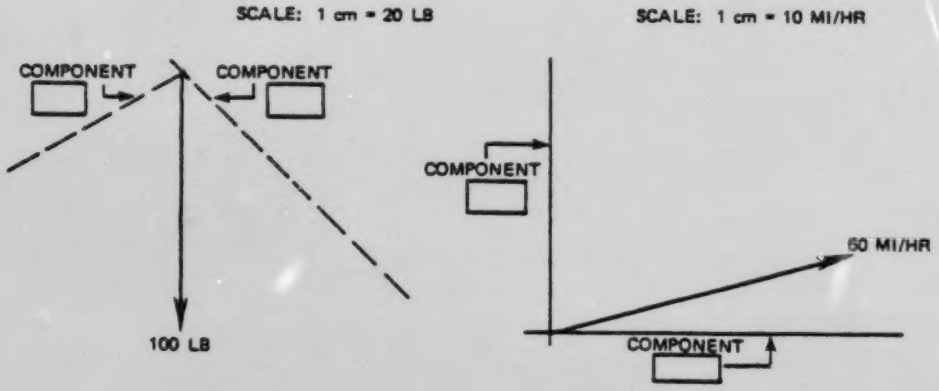
11. On a piece of graph paper, using protractor and scale, work the following problem:

Explorers leave their base and travel the following distances on three succesive days:

1st day: 60 miles – 50° East of North.
2nd day: 40 miles – 90° East of North (due East).
3rd day: 50 miles – 120° East of North.

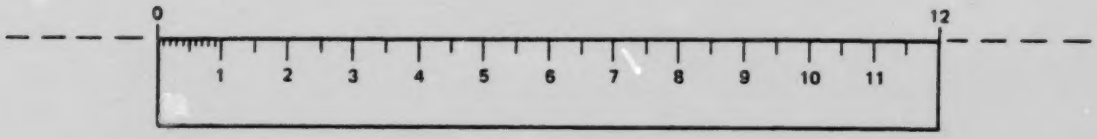
How far and in what direction must the explorers travel in order to return to their base on the fourth day?

12. In each of the vectors illustrated below, sketch graphically their components, and find the magnitudes of each component.



13. Repeat Problem 11 using the mathematical solution method. Compare your answer to that obtained in Problem 11.
14. Repeat problem 2 using the mathematical solution method.

NOTE: Because of reduction in copy size caused by printing methods used in reproducing these course materials, the scale of the drawings in this module may vary slightly. To ensure consistency in the measurement of vector quantities in this module which have been drawn to scale, it is essential that the following ruler be used as the measuring standard.



Ruler

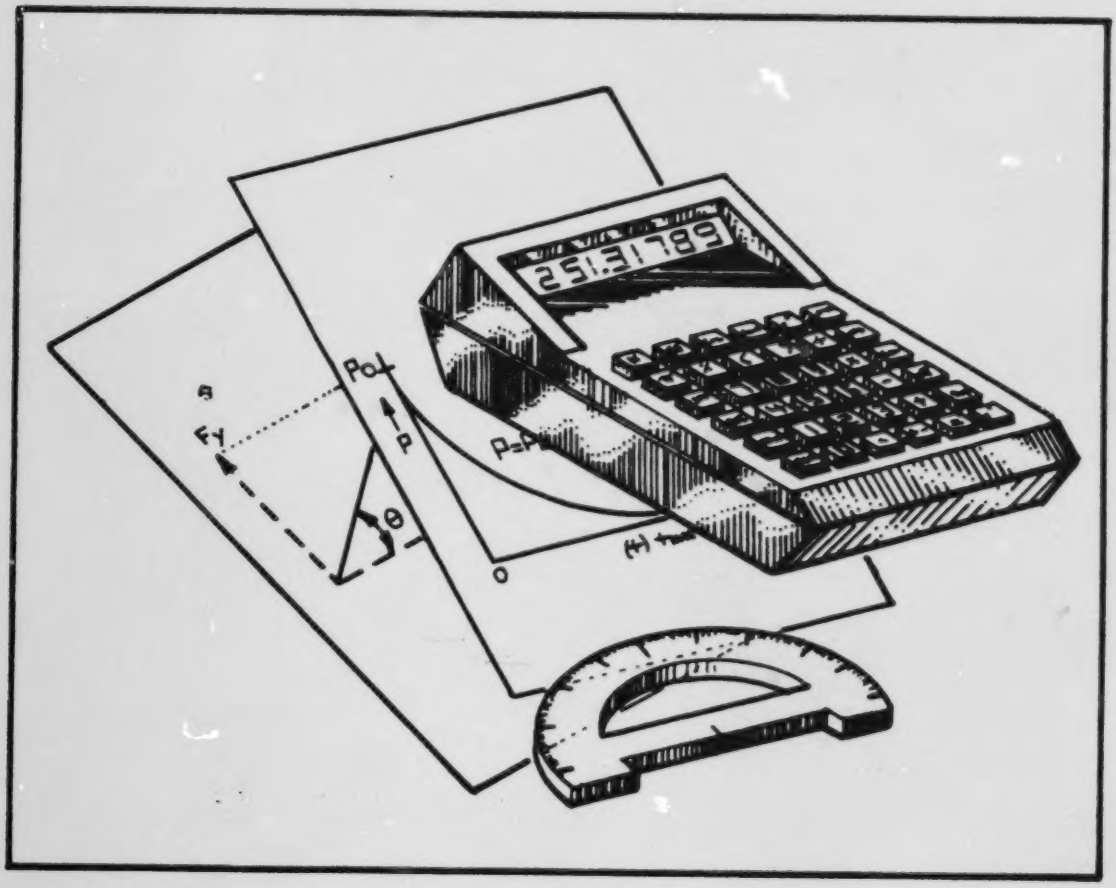
Recommend that the student remove this sheet from the module, fold the sheet on the above dotted line, and use the upper edge of the ruler in making vector length measurements.

UTC

UNIFIED TECHNICAL CONCEPTS

MODULE S 6

READING AND DRAWING GRAPHS



SUPPLEMENTARY

CORD

CENTER FOR OCCUPATIONAL RESEARCH AND DEVELOPMENT

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INTRODUCTION

Technical and scientific work often is a study of the relationship between two interacting physical quantities. If one quantity is varied, the other quantity is affected. A graphical display of the values of two physical quantities shows clearly how one quantity is related to another. Incorrect data values are detected more easily when observed in graphical form than when seen as a table of values. A graph of experimental data serves frequently as a communication device and as a tool for evaluation of technical data by scientists, engineers, and technicians. For the solution of many technological problems, the graphical method is sufficiently accurate, faster than algebraic methods, and less susceptible to error.

In this module, the student will be presented a proper and generally-acceptable method of preparing data in graphical form. The student will examine the criteria of acceptability for graphs.

OBJECTIVES

Upon completion of this module, the student should be able to draw a technically-acceptable graph from a table of experimental data suitable for plotting on rectangular-coordinate graph paper. The resulting graph may be linear or nonlinear.

BASIC STEPS FOR GRAPHING EXPERIMENTAL DATA

1. Examine the tabular data, and identify which variable is the independent variable and which is the dependent variable. In the experiment, the values of the independent variable are set, or controlled, by the technician or experimenter. The dependent variable is measured as a result of setting a particular value of the independent variable. In graphing practice, the independent variable customarily is plotted along the abscissa (horizontal axis), and the dependent variable along the ordinate (vertical axis), as shown in Figure 1.

   Sometimes, by examination of the incremental steps in the values of the variables, the independent and dependent variables can be identified. The independent variable often will be incremented in even steps, and the dependent variable values will appear to have a random incrementation, as shown in Figure 2.

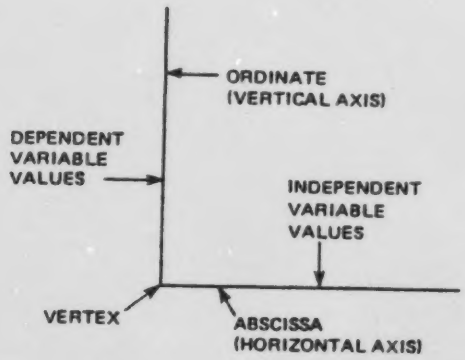


Figure 1. Graph coordinate axis.

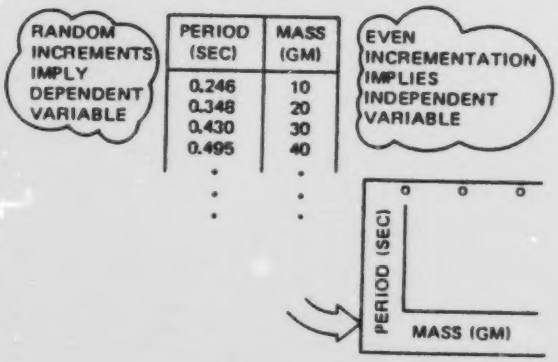


Figure 2. Recognition of independent/dependent variables.

In this module, the exercises will aid in the identification of independent and dependent variables by identifying them in a customary way. In any exercise, the variable mentioned first in the instruction problem is the dependent variable, and the second variable mentioned is the independent one. An example is, "Plot the variation of voltage with

current." Here, voltage is the dependent variable and current is the independent variable.

2. Select standard-quality, commercial graph paper.
3. Select the position of the paper for which the rectangular coordinate axes will be drawn. Acceptable positions are illustrated in Figure 3. The holes of the paper, if any, should be on the left hand side of the paper or at the top of the page.

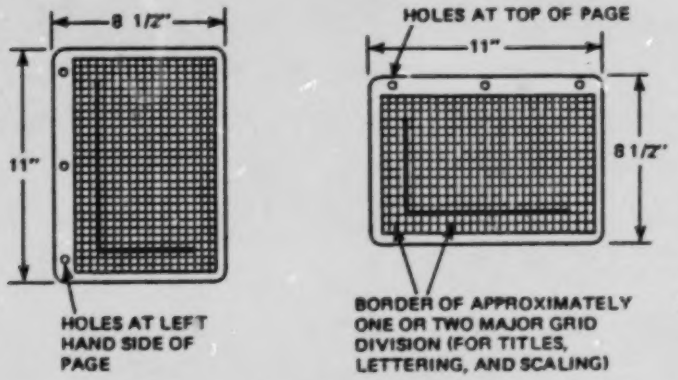

8 1/2"
11"
HOLES AT LEFT HAND SIDE OF PAGE
HOLES AT TOP OF PAGE
11"
8 1/2"
BORDER OF APPROXIMATELY ONE OR TWO MAJOR GRID DIVISION (FOR TITLES, LETTERING, AND SCALING)

Figure 3. Orientation and format of graph paper.

The axes are drawn as shown in Figure 3, leaving a space of one or two major grid divisions from the border. This space will used for title and scale lettering.

4. For each variable, select a scale that conveys the best information to the best advantage. The choice of scales is the most important and, at the same time, the most difficult part of graph construction. Scale selection does more to shape the picture of the variable relationship than all other steps; it makes the difference between an accurate picture and a misleading one. For this reason, the discussion of scales must be somewhat lengthy; however, an attempt is made to prevent many common student errors before they occur. Some helpful hints are as follows:

   - Make a trial computation to select the scale length that will "fit" on the page and will have divisions easy to plot. Choose a length that will spread the data as much as possible along the axis on which the variable is to be plotted, while at the same time arranging that the units per division are 1, 2, 5, 10, and sometimes 4 or multiples of these numbers. Never use 3, 7, or 9, since these values would cause a graph to be difficult to plot and to read.

- The divisions on the vertical axis need not be the same as those on the horizontal axis. Each scale should be chosen independently as described previously.

- Construct the axes and scales in such a manner that all quadrants required to display the positive and negative ranges of the data are available.

- The lettering of intermediate scale values over the scale range requires some care for clarity. Scale values always increase away from the lower left-hand corner of a first-quadrant graph; that is, upward or to the right. Intermediate scale values should be chosen for ease of rapid location of data points and should not be crowded together. Some common errors are shown in Figure 4.

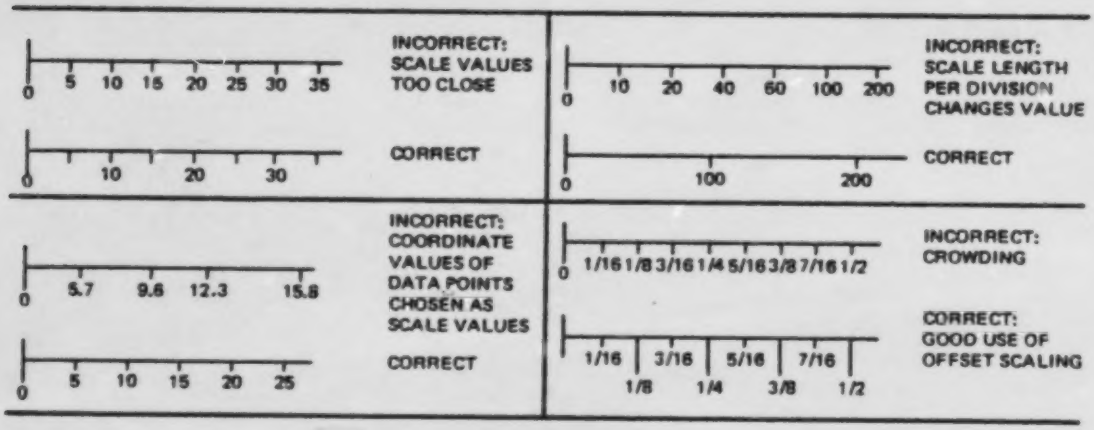

0 5 10 15 20 25 30 35
INCORRECT: SCALE VALUES TOO CLOSE
0 10 20 30
CORRECT
0 10 20 40 60 100 200
INCORRECT: SCALE LENGTH PER DIVISION CHANGES VALUE
0 100 200
CORRECT
0 5.7 9.8 12.3 15.8
INCORRECT: COORDINATE VALUES OF DATA POINTS CHOSEN AS SCALE VALUES
0 5 10 15 20 25
CORRECT
0 1/16 1/8 3/16 1/4 5/16 3/8 7/16 1/2
INCORRECT: CROWDING
1/16 3/16 5/16 7/16
1/8 1/4 3/8 1/2
CORRECT: GOOD USE OF OFFSET SCALING

Figure 4. Selection and labeling of scale values.

- Print the numerals for the scale, using the coordinate lines of the graph paper as lettering guides. The numerals should be 2 or 3 divisions high, depending upon the coordinate paper. Center the numerals about the scale coordinates that they identify.

- Zero is the most desirable beginning of the ordinate and abscissa axes unless this would compress the curve unnecessarily. The origin is placed in the lower left-hand corner except for cases in which both positive and negative values are to be plotted. In such cases, the origin should be located in order that all desired values can be shown. Frequently, the intersection of the two axes in the left-hand corner of the graph is labeled with "even, chosen" values slightly less than the lowest values in the ranges of the two variables, as shown in Figure 5.

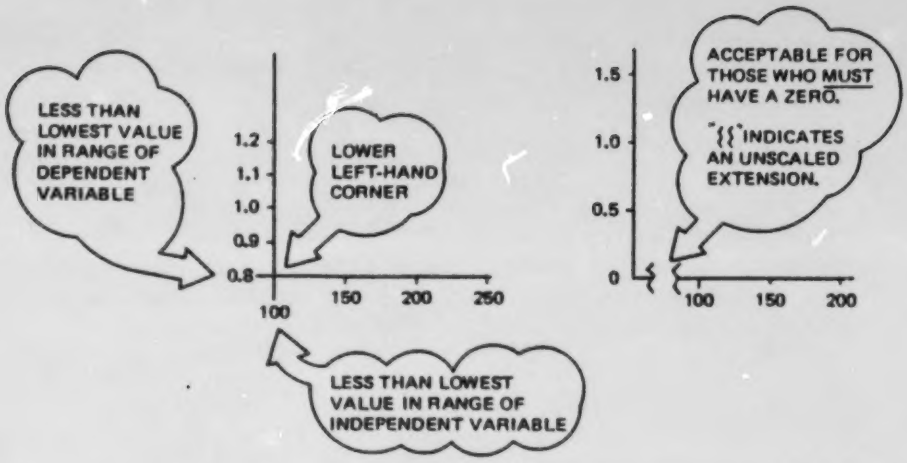


Figure 5. Selecting the beginning of the ordinate and abscissa.

- In the space outside the scale, print a clear, complete label. Letter each scale on the graph so that the scale label can be read from the lower left-hand corner. A good practice is to write all labels in capital letters, although some authorities capitalize only the first word and proper names. In either case, a consistent style should be chosen. The scale label includes the name of the quantity plotted and its unit, separated by a comma or a parenthesis. Abbreviate all units in standard form. Figure 6 shows examples of scale labeling.

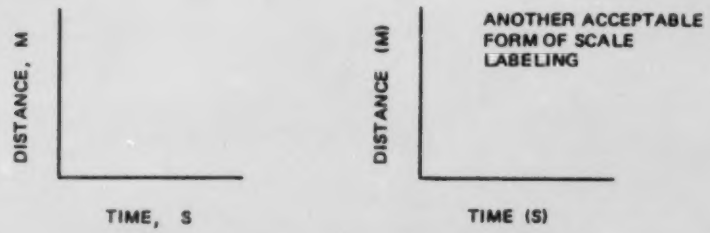


Figure 6. Labeling the ordinate and abscissa.

5. LOCATION OF DATA POINTS. Locate each data point in its proper place with a fine, tiny pencil dot, and draw a circle not more than 1/16 inch in diameter around the point. When multiple curves are plotted on one sheet, the points for each curve may be identified by other symbols, such as squares, diamonds, or other suitable geometric figures.
6. DRAWING THE CURVE. When plotted, experimentally-determined data often show a scattering of points about some smooth curve drawn through the

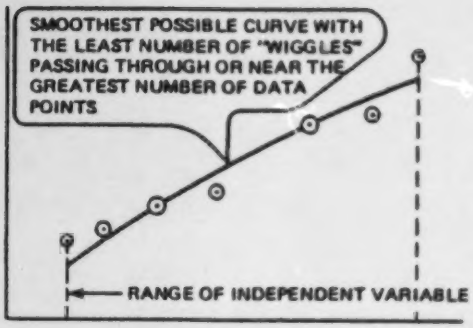

SMOOTHEST POSSIBLE CURVE WITH THE LEAST NUMBER OF "WIGGLES" PASSING THROUGH OR NEAR THE GREATEST NUMBER OF DATA POINTS
RANGE OF INDEPENDENT VARIABLE

Figure 7. Fitting a smooth curve to experimental data.

points, as shown in Figure 7. In experimental work, points usually do not fall exactly upon a smooth curve, because of errors and inaccuracies that exist in the measurement process. A light, penciled, free-hand line will aid in locating the average; but the final line should be drawn smoothly and clearly with a flexible curve or other drawing aid. In drawing the final curve, do not draw the line through the symbols that enclosed the plotted points. Stop at the perimeter of each symbol (i.e., _____). Do not extend the curve beyond the range of the plotted data points.

7. TITLE OF GRAPH. The title of the graph should include the names of plotted quantities and should include other descriptive information, such as the value of relevant quantities held fixed during the experiment, names of equipment, date the data was obtained, where the data was obtained, and any other descriptive information about the graph. (See Figure 8.)

   The title should be placed in such a position on the sheet that it will not interfere with the curve. The title section of a display graph usually is placed across the top of the sheet. Simple graphs that comprise parts of reports frequently have the title in either the lower right quadrant or the upper left quadrant.

8. Finally, the name of the person preparing the graph and the date that the graph is plotted should be placed in the lower right-hand corner of the sheet.

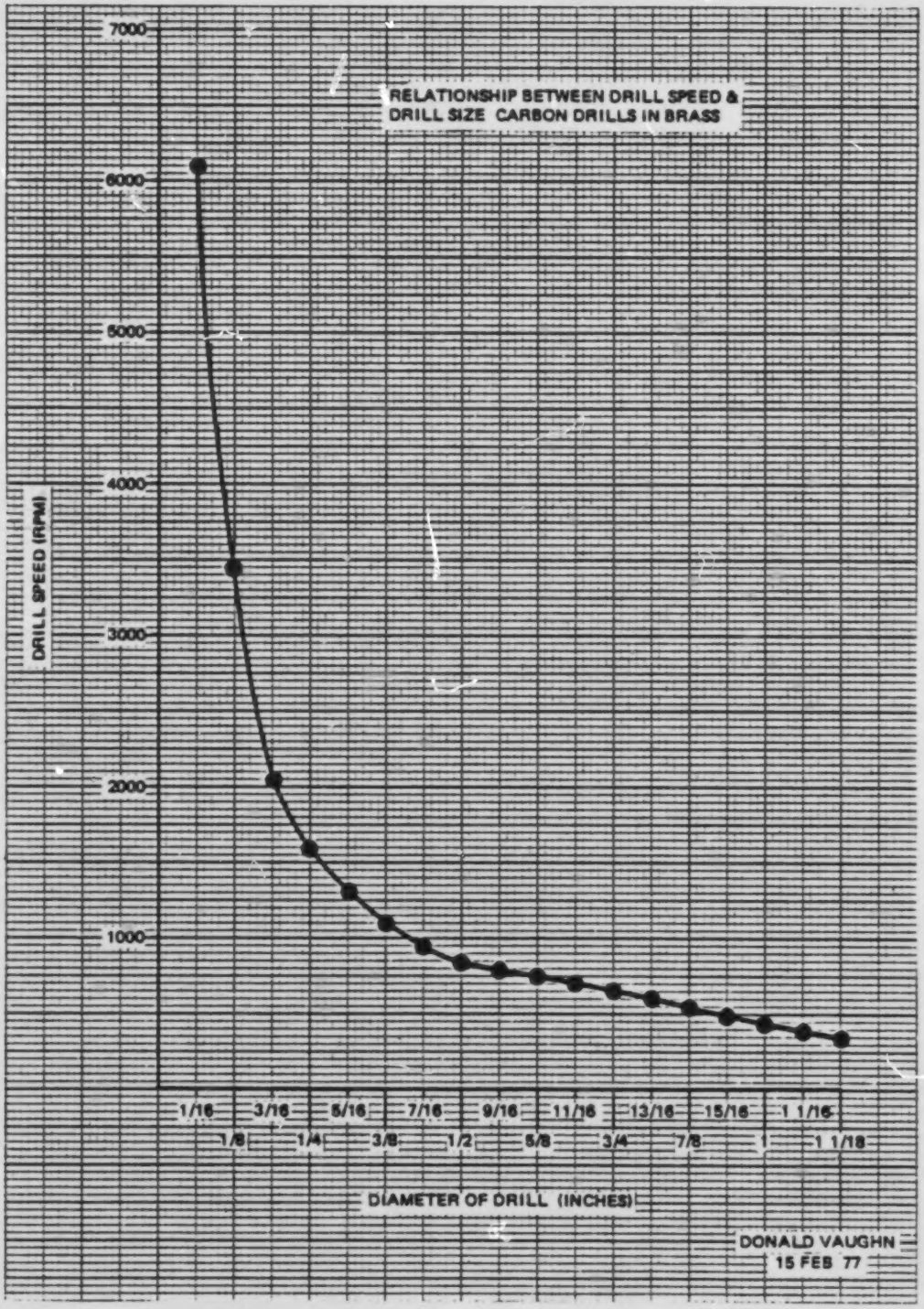


Figure 8. A model graph

SUMMARY

Graphs demonstrate how one quantity is related to another. The visual display furnished by graphs simplifies many relationships that might be less obvious when stated in equation form. The graphical solution generally is sufficiently accurate and, in addition, is less susceptible to error than algebraic methods.

The technician will have opportunity to use graphs in (1) plotting data and in (2) interpreting the graphical data presented by others in reports, manuals, textbooks, articles, etc.

REFERENCES

Aldrige, Bill G. Quantitative Aspects of Science and Technology: An Introduction. Columbus, OH: Charles Merrill Publishing Co., 1967.

Giesecke, Frederick E.; Mitchell, Alva; Spencer, H. C.; and Hill, I. L. Technical Drawing. 5th ed. New York: The Macmillan Company, 1967.

Meyer, Stuart L. Data Analysis for Scientists and Engineers. New York: John Wiley & Sons, Inc., 1975.

Richard, Paul S. and Shaevel, Leonard M. Essentials of Technical Mathematics. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1974.

REVIEW QUESTIONS

MATERIALS

One package of rectangular coordinate graph paper, 10 x 10 to the inch or 10 millimeters to the centimeter. (8 1/2 x 11.)

One flat plastic ruler (1 foot long.)

One No. 2 pencil with eraser.

One black ballpoint or fine black felt-tip pen.

One flexible curve drawing aid or a set of French curves.

From the following set of graph problems, choose any two review exercises and correctly present the data in graphical form using the following checklist:

1. Graph will be produced on one side of graph paper only.
2. Graph should be a full page presentation, with graph occupying approximately 3/4 of page area.
3. Final copy is prepared totally in ink. Location of data points and drawing of curve can be sketched preliminarily with light pencil.
4. The independent variable is scaled on abscissa; dependent variable is scaled on ordinate.
5. The scaling of the axis should be —
   a. Chosen to encompass range of the variable.
   b. Chosen in least-count increments of 1, 2, 5, or 10 square widths per unit for ease of data plotting.
   c. Numbered neatly, without crowding, and in the same constant increments (for linear scale).
   d. Chosen in such a manner that the length of scale is neither unduly compressed (too short), possibly masking the behavior of the data, nor unduly elongated (too lengthy), possibly leaving too little space for all data points to be plotted.
   e. Arranged in such a manner that at least two significant figures can be read from the scale.
6. Coordinate axes are labeled with variable names (not symbols), followed by a comma and by the unit (may be abbreviated). Use grid lines as lettering guides, and use capital letters (2 or 3 squares high) for 10 x 10 to inch grid.

7. Data points are located by small circles drawn around the data points. The diameter of these circles should be large enough to be seen easily without exceeding 0.4 cm.
8. The smoothest possible curve is drawn through the vicinity of the data points in order that as many points as possible fall on or near the curve. Curves should not be drawn through the data-point symbols.
9. The title of the graph should include the names of the plotted quantities and should include other suitable information that would help describe the graph.
10. The title should be placed on the sheet, but should not interfere with the curve. The title section of graphs frequently is placed across the top or bottom of the graphing sheet.
11. The general overall appearance should be neat, orderly, and easy to read and to interpret.
12. Why is a graph usually interpreted more easily than an equation?
13. Plot a graph showing the relationship of normal barometric pressure of air-to-altitude. Plot values up to and including 15,000 ft. (Suggestion: Plot the independent variable along the longer length of graph paper.)

Altitude (Feet above Sea Level)	Normal Barometric Pressure (Inches of Mercury)
0	29.95
500	29.39
1000	28.86
1500	28.34
2000	27.82
2500	27.32
3000	26.82
4000	25.84
5000	24.9
6000	24.0
7000	23.1
8000	22.2
9000	21.4
10000	20.6
15000	16.9

14. The following data was taken in the laboratory for a 75-watt, gas-filled, tungsten-filament light bulb. Plot a resistance-voltage curve.

Voltage (volts)	Resistance (ohms)
10	47.5
20	77.5
30	100.3
40	119.0
50	132.6
60	144.2
70	160.2
80	170.0
90	178.3
100	189.0
110	200.1

15. Plot a graph showing the variation of temperature with electric current through a heating coil. The following data were taken in the laboratory:

Current (amp)	Temperature (°Celsius)
0.0	0.0
0.46	0.5
1.05	1.2
1.50	2.0
2.06	5.1
2.20	7.7
2.35	8.8

16. For a fairly clean iron pipe, 1 inch in diameter, and water at a temperature of about 70°F, the following losses of head pressure in inches of water per 1000 feet of pipe were calculated for water velocities ranging from 6 inches per second to 30 inches per second.

Velocity of Water in Pipe (Inches per Second)	Loss of Head Pressure (Inches of Water per 1000 Feet)
6	20
12	80
18	180
24	300
30	440

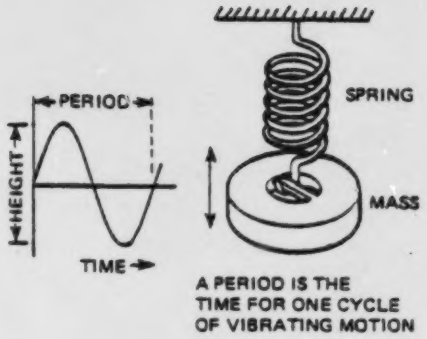
Plot a graph of the loss of head pressure versus water velocity for this pipe.

17. Plot the relation between magnetic flux density (B) in Kilolines per square centimeter and magnetizing force (H) in Gilberts per centimeter for a specimen of tool steel. The graph will form what customarily is called a "B-H curve."

Magnetic Flux Density (B) (Kilolines per cm^2)	Magnetizing Force (H) (Gilberts per Centimeter)
9.00	27.1
11.80	54.2
13.02	81.3
13.75	108.4
14.09	135.5
14.22	162.6
14.66	189.7
14.86	216.8
14.98	243.9
15.23	271.0
15.35	298.1
15.57	325.2

18. Plot a graph for the following experimental data, showing the relation between the Period in seconds and the Mass in grams of a vibrating spiral spring.

Period (Seconds)	Mass (Grams)
0.246	10
0.348	20
0.430	30
0.495	40
0.570	50
0.650	70
0.740	90
0.810	110
0.900	130
0.950	150



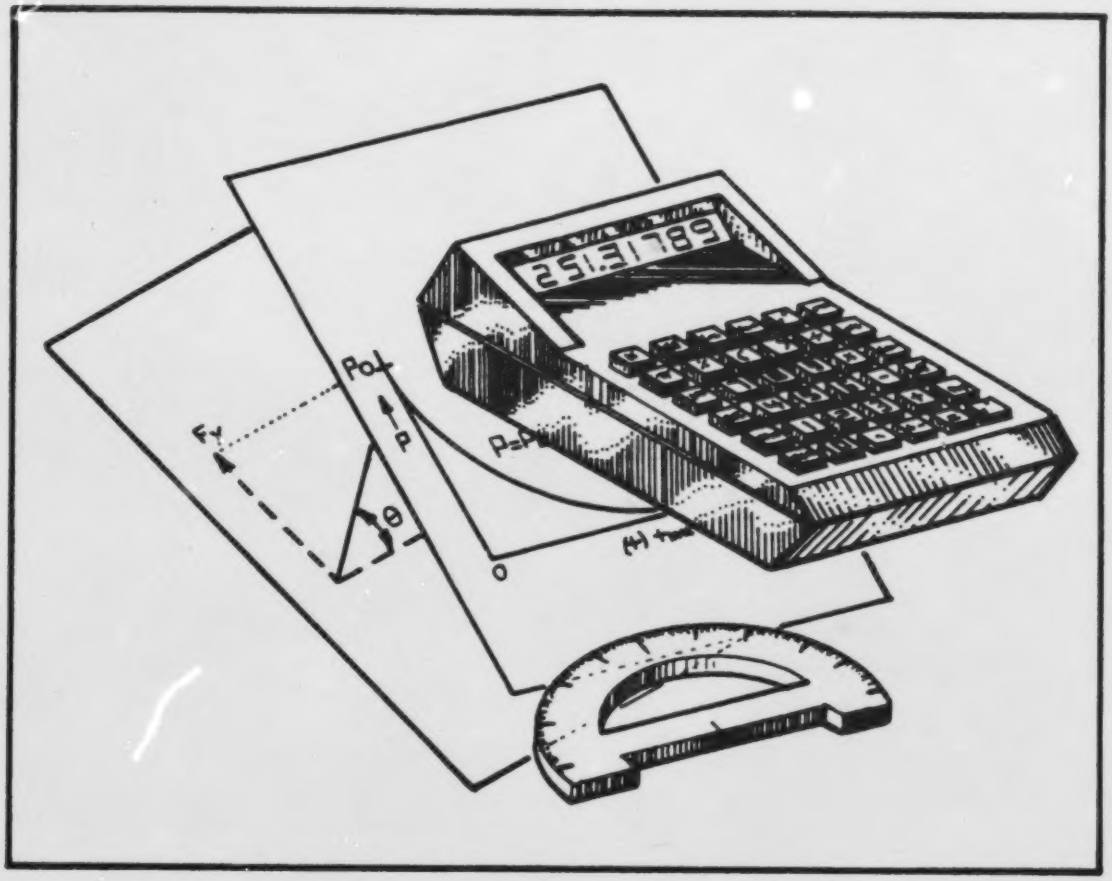
A PERIOD IS THE TIME FOR ONE CYCLE OF VIBRATING MOTION

UTC

UNIFIED TECHNICAL CONCEPTS

MODULE S 7

LOGARITHMS AND EXPONENTS



SUPPLEMENTARY

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INTRODUCTION

Scientists, engineers, and technicians work with data which range from astronomically large numbers to extremely small numbers. To express these large and small numbers efficiently, a particular notation has been adopted. This notation — known as "exponential," or "power-of-ten," notation — facilitates the recording and manipulating of numerical data.

Logarithms are important in engineering, chemistry, physics, electronics, and other fields because many problems involving growth and decay common to these fields are "exponential" (logarithmic in nature). In this age of computers and electronics, difficult calculations often are done by computer. The availability of electronic calculators and computers, however, has not decreased the importance of understanding the basic tools in carrying out calculations that involve mathematical data. Embarrassment and incorrect answers can result when engineers or technicians unfamiliar with logarithmic manipulations are forced to use logarithms (perhaps because of a power failure of the calculator!) Furthermore, an engineer or technician may struggle through a laborious method of solving a problem which could have been solved easily with logarithms.

In this module, the principles of power-of-ten exponents and logarithms will be presented so that the student can apply these principles to specific calculations. "Natural logarithms" and exponents of the base "e" are also described, as well as logarithmic scales on graphs.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Given a number in decimal form, express it in scientific notation. Given a number in scientific notation, express it in decimal form.
2. Add, subtract, multiply, or divide, using power-of-ten numbers.
3. Find logarithms and antilogarithms of given numbers, using tables and linear interpolation for common logarithms and natural logarithms.
4. Using the three theorems of logarithms for products and quotients, multiply and divide numbers and solve equations.

5. Given an equation containing logarithmic or exponential terms and the numerical values of all but one variable in the equation, compute (calculate) the value of the unknown variable with the aid of log tables or an electronic calculator.

SUBJECT MATTER

According to the Guiness Book of World Records, the remotest astronomical body that can be seen with the naked eye is in the galaxy Andromeda nebula — a distance of some 13,000,000,000,000,000,000 miles. Frequently, large and small numbers, which would require a great amount of space and effort to write in decimal notation, are encountered in technical work. The same distance mentioned previously can be written more compactly in an exponential form of notation called "power-of-ten." The distance would be written 1.3×10^{19} miles. Representing the distance in this manner not only makes writing large and small numbers easier, but also simplifies calculations and reduces the amount of space required to report such manners.

In order to rewrite a decimal number in power-of-ten form, the number must be factored into two separate parts, just as 80 could be separated into a product of the factors, 8.0 and 10, i.e. (8.0 x 10). The first leading factor contains the significant digits of the number. Significant digits are the specified digits of a number; they indicate the accuracy of the number or measurement. The second factor following the multiplication sign (x) is a number that contains the power of ten. The power of ten contained in the second number is expressed with a small number written slightly above and to the right of the number 10. This number is called the "exponent," an example of which is illustrated in Figure 1. Two cautions should be observed when converting a number from decimal notation to power-of-ten, or vice versa. When converting from one notation to the other, no rounding off, or dropping of

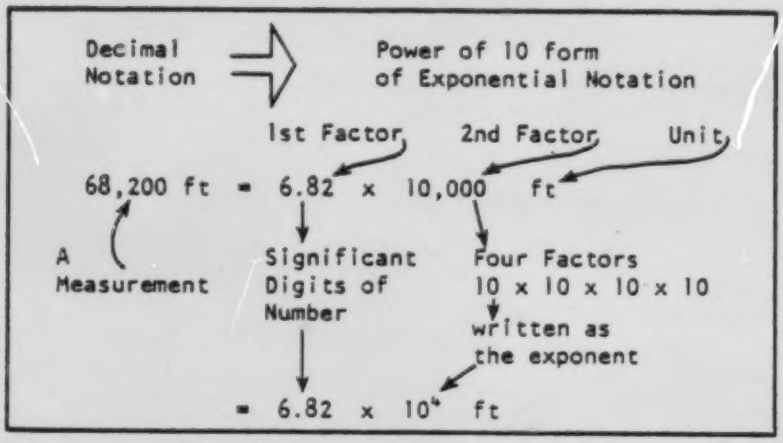
Decimal Notation
Power of 10 form of Exponential Notation
1st Factor
2nd Factor
Unit
68,200 ft = 6.82 x 10,000 ft
A Measurement
Significant Digits of Number
Four Factors 10 x 10 x 10 x 10
written as the exponent
= 6.82 x 10⁴ ft

Figure 1. Expressing a number in scientific notation.

significant digits, should occur. The accuracy of a measurement should not change because of a change in notation in reporting the numbers. Although not a fixed rule, the first factor customarily is represented as a number between 1 and 10. In this practice, the number is expressed in a special kind of exponential notation using power-of-ten called "scientific notation."

Table 1 lists several examples of powers-of-ten and their decimal equivalents. A review of this table is worthwhile before proceeding to examples of conversion of numbers from decimal notation to power-of-ten. Numbers greater than 1 will have exponents of positive algebraic sign, and numbers less than 1 will have exponents with negative algebraic signs.

TABLE 1. POWER-OF-TEN AND DECIMAL EQUIVALENTS.

Power of 10	Decimal Equivalent	Explanation
10^5	100,000	10 x 10 x 10 x 10 x 10
10^3	1,000	10 x 10 x 10
10^1	10	(10^1 is usually expressed as 10 without showing the exponent)
10^0	1	By definition — any number to the zero power equals 1.
10^{-1}	0.1	$\frac{1}{10}$
10^{-2}	0.01	$\frac{1}{10 \times 10}$
10^{-3}	0.001	$\frac{1}{10 \times 10 \times 10}$

Every decimal number can be expressed in power-of-ten notation. Consider the following examples:

$$375{,}000 = 3.75 \times 10^5$$
$$81{,}000 = 8.1 \times 10^4$$
$$623{,}000{,}000{,}000 = 6.23 \times 10^{11}$$
$$0.0715 = 7.15 \times 10^{-2}$$
$$0.000000133 = 1.33 \times 10^{7}$$

All the above examples have been written in scientific notation. Other powers-of-ten also can be used to represent the magnitude of numbers; however, the first factor must be adjusted by a factor of ten. This point is made clear by the second example above, which can be written in several powers-of-ten:

power-of-ten exponential form

$$\begin{cases} 81{,}000 = 8.1 \times 10^4 & \text{(scientific notation)} \\ = 81 \times 10^3 \\ = 180 \times 10^2 \\ = 8100 \times 10^1 \\ = 0.81 \times 10^5 \end{cases}$$

In calculation with power-of-ten numbers, many instances occur in which one or more numbers might be represented best in a power-of-ten other than scientific notation.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF NUMBERS EXPRESSED IN POWERS OF TEN

The following rules must be observed when mathematical operations are conducted with numbers expressed in power-of-ten form:

Rule 1: Any number to the zero power (exponent) is equal to 1.

Examples: $10^0 = 1$, $2^0 = 1$, $5^0 = 1$

Rule 2: Any number to the first power is equal to that number.

Examples: $10^1 = 10$, $2^1 = 2$, $5^1 = 5$

Rule 3: If n is a positive number and is a non-zero number, then

$$a^{-n} = \frac{1}{a^n}$$

Example: $10^{-2} = \frac{1}{10^2}$

Rule 4: Before numbers are added or subtracted, the exponents of ten must be the same. The first factor need not be between 1 and 10.

Example 1:

$$\begin{array}{rrl} & 2.610 \times 10^{3} & = 2.610 \times 10^{3} \\ \text{Add } (+) & \underline{6.2000 \times 10^{4}} & \underline{= 62.000 \times 10^{3}} \\ & & 64.61 \times 10^{3} \end{array}$$

or

$$6.461 \times 10^{4}$$

Example 2:

$$\begin{array}{rrl} & 3.930 \times 10^{-6} & = 3.930 \times 10^{-6} \\ \text{Subtract } (-) & \underline{7.81 \times 10^{-7}} & \underline{= 0.781 \times 10^{-6}} \\ & & 3.149 \times 10^{-6} \end{array}$$

Rule 5: In multiplication, the first factors (decimal part) of each number are multiplied, but the exponents of 10 are added algebraically. The result is a product of the first factors times (x) a power of ten.

Example 1: $(3.2 \times 10^{4})(1.5 \times 10^{2})$

$= (3.2)(1.5) \times 10^{4+2}$

$= 4.8 \times 10^{6}$

Example 2: $(3.2 \times 10^{-3})(1.5 \times 10^{2})$

$= (3.2)(1.5) \times 10^{-3+2}$

$= 4.8 \times 10^{-1}$

Rule 6: In division, the first factors of each number are divided, but the exponents of 10 are subtracted algebraically. The result is the quotient of the first factors times (x) a power of ten.

Example 1: $\dfrac{8.4 \times 10^{9}}{2.1 \times 10^{4}} = \dfrac{8.4}{2.1} \times 10^{9-4}$

$= 4.0 \times 10^{5}$

Example 2: $\dfrac{8.4 \times 10^{7}}{2.1 \times 10^{-3}} = \dfrac{8.4}{2.1} \times 10^{7-(-3)}$

$= 4.0 \times 10^{10}$

LOGARITHMS

A common logarithm (log) is the exponential expression of a number having 10 as the base; that is, it is the power-of-ten required to produce that number. Log_b (y) means "the power of b to produce y." When the base is not given, it is implied to be 10. An example of an equation written in exponential form is $y = b^x$. Written in logarithmic form, this equation gives $x = \log y$. Figure 2 contrasts the two different forms of equations and illustrates the basic definitions in each form.

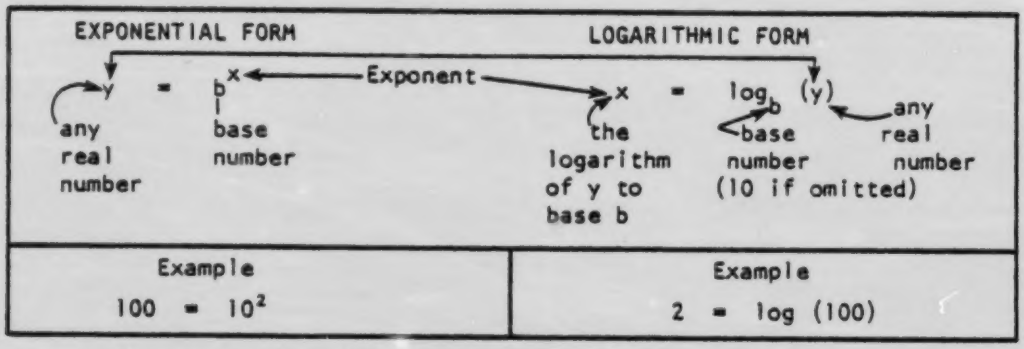

Figure 2. Converting numbers to logarithms and exponents.

Figure 3 illustrates several exponential and logarithmic forms of equations having 10 as the base. The logarithms of numbers between 1.00 and 9.99 are given in Table 2, a Table of Common Logarithms of Numbers.

To find the logarithm of any number from Table 2, locate the first two digits of the number in the left hand column of the table. Next, move horizontally to the right until the column headed by the third digit of the number is encountered. The desired logarithm of the number lies at this intersection in the table. It is expressed as a decimal fraction less than 1. Example A illustrates the method of finding a logarithm in Table 2.

EXPONENTIAL FORM		LOGARITHMIC FORM		
1	$= 10^{0.00000}$	log	1	= 0.00000
2	$= 10^{0.3010}$	log	2	= 0.3010
3	$= 10^{0.4771}$	log	3	= 0.4771
4	$= 10^{0.6021}$	log	4	= 0.6021
5	$= 10^{0.6990}$	log	5	= 0.6990
6	$= 10^{0.7882}$	log	6	= 0.7882
7	$= 10^{0.8451}$	log	7	= 0.8451
8	$= 10^{0.9031}$	log	8	= 0.9031
9	$= 10^{0.9542}$	log	9	= 0.9542
10	$= 10^{1.0000}$	log	10	= 1.0000
100	$= 10^{2.0000}$	log	100	= 2.0000
1000	$= 10^{3.0000}$	log	1000	= 3.0000
0.1	$= 10^{-1}$	log	0.1	= -1
0.01	$= 10^{-2}$	log	0.01	= -2
0.001	$= 10^{-3}$	log	0.001	= -3

Figure 3. Base 10 logarithms and exponents.

TABLE 2. COMMON LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0034	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5623	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	729[illegible]	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
N	0	1	2	3	4	5	6	7	8	9

TABLE 2. Continued.

N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8765	8781	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
N	0	1	2	3	4	5	6	7	8	9

EXAMPLE A: FINDING THE LOGARITHM OF A NUMBER.

Given: Table 2, Common Logarithms of Numbers.

Find: The logarithm of 1.64.

Solution: Locate the digits 16 in the lefthand column of Table 2 as shown below. Move horizontally to the right until the column headed by the 4 is intersected. The digits at the intersection are 2148, which represent the decimal fraction less than 1 and the logarithm of 1.64.

In summary: log (1.64) = 0.2148

Portion of Table 2

Table 2. COMMON LOGARITHMS OF NUMBERS

N	0	1	2	3	(4)	5	6	7	8	9
10	0000	0043	0086	0128	0[illegible]0	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0[illegible]9	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0[illegible]4	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1[illegible]7	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1[illegible]4	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	[illegible]	[illegible]	[illegible]	[illegible]	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989

To find the logarithms of numbers greater than 10, the numbers must be converted to scientific notation, and use must be made of a property of logarithms that states that "the logarithm of the product of two numbers is equal to the sum of the logarithms of the individual numbers." For instance, the logarithm of 16.4 is found when 16.4 is converted to 1.64×10^1, as witnessed in Example B.

EXAMPLE B: FINDING THE LOGARITHM OF A NUMBER GREATER THAN TEN.		
Given:	Table 2, Common Logarithms of Numbers.	
Find:	Log (16.4).	
Solution:	$\log (16.4) = \log (1.64 \times 10^1)$	
	$= \log (1.64) + \log (10^1)$	
	$= 0.2148 + 1.000$	
	$\log (16.4) = 1.2148$	
which means that,	$10^{1.2148} = 16.4$	in exponential form
likewise,	$\log (164) = \log (1.64 \times 10^2)$	
	$= \log (1.64) + \log (10^2)$	
	$\log (164) = 2.2148$	
or equivalently,	$10^{2.2148} = 164$	

The logarithm of the power-of-ten is called the "characteristic," and the logarithm of the first factor of a number containing the significant digits in scientific notation is called the "mantissa." The mantissa is the logarithm located in the table of common logarithms; therefore, the logarithm of any number greater than 10 is the sum of a mantissa and a characteristic.

Negative logarithms frequently cause difficulties. Recall that power-of-ten numbers less than 1 have negative exponents ($0.01 = 10^{-2}$). To determine logarithms of numbers between 0 and 1, write the number in standard scientific notation; find its mantissa in the log table; and supply its negative characteristic. This procedure is clarified in Example C.

EXAMPLE C: FINDING THE LOGARITHM OF A NUMBER LESS THAN ONE.	
Given:	Table 2, Common Logarithms of Numbers.
Find:	The logarithm of 0.0265.
Solution:	1. Write 0.0265 as 2.65×10^{-2}.
	2. Find the mantissa in the log table. Its mantissa is 0.4232.
	3. Its characteristic is -2 (a negative exponent).

Therefore: $\log (0.0265) = 0.4232 + (-2) = 0.4232 - 2$
$= -1.5768$

FINDING THE ANTILOGARITHM

The number corresponding to a given logarithm often must be found. As an example, find x, given that $\log x = 0.5999$. Since the characteristic is 0, x must be a number between 1 and 10. The digits 5999 in Table 2 are located in such a manner so as to correspond to 39 in the left-hand column (laterally) and to the number 8 heading in the vertical column; therefore, $x = 3.98$ ($\log [3.98] = 0.5999$).

If y is the logarithm of a number x, then x is called the "antilog" of y, as in Example D.

EXAMPLE D: FINDING THE ANTILOG OF A NUMBER.

Given: Table 2, Common Logarithms of Numbers.

Find: a. The antilog of 2.9547.

b. The antilog of -4.3655.

Solution: a. From the table of common logarithms,

Antilog (0.9547) = 9.01

Then

Antilog (2.9547) = 9.01×10^2
= 901

b. Most tables are constructed for positive mantissas only. To find the antilog of a negative number using such a table it is necessary to first express the number as the sum of two numbers. One of these is the postive matissa and the other is the negative characteristic:

Antilog (-4.3655) = Antilog [(5 - 4.3655) - (5)]

(5 is added to and subtracted from the same term preserving the equality)

Antilog (-4.3655) = Antilog (0.6345 - 5)

positive mantissa; negative characteristic

Applying the basic theorem of logarithms

$$\log (x + y) = (\log x)(\log y)$$

$$\text{Antilog } (-4.3665) = [\text{Antilog } (0.6345)] \times [\text{Antilog } (-5)]$$

$$= 4.31 \times 10^{-5}$$

Antilog (-4.3655) = 0.0000431

The reasoning for the first step in Example D is the fact that mantissas always must be positive numbers, although characteristics may be negative or positive. If -4.3655 had been represented as (-4-.3655), then -0.3655 could not be the mantissa, because it is not a positive number as required.

INTERPOLATION

To find the logarithm of a number that lies between two given numbers in Table 2, one can assume a linear change in numbers between the nearest logarithms and interpolate, as shown in Example E.

EXAMPLE E: INTERPOLATION.

Given: Table 2, Common Logarithms of Numbers.

Find: log 247.2.

Solution:

log 248 = 2.3945

and log 247 = 2.3927

difference = 0.0018

Then, log 247.2 = 2.3927 + (0.2) x (0.0018)

= 2.3927 + 0.00036

log 247.2 = 2.39306 ≈ 2.3931

EVALUATING FORMULAS CONTAINING LOGARITHMS

The preceding discussion of the mechanics of obtaining logarithms is a necessary prerequisite to any evaluation of formulas containing logarithms. One such formula is the equation for gain used in the study of electrical power, light energy, and sound energy. Its form is given below:

$$D = 10 \log \left(\frac{P_2}{P_1}\right)$$

In this formula, $\log (P_2/P_1)$ is the logarithm of a fraction. If the values of P_2 and P_1 are known, the corresponding value of D can be found as shown in Example F.

EXAMPLE F: SOLVING LOGARITHMIC EQUATIONS.

Given: $D = 10 \log (P_2/P_1)$.

$P_2 = 15.5$, and $P_1 = 12.7$.

Find: D.

Solution:

$$D = 10 \log \left(\frac{15.5}{12.7}\right)$$

$D = 10 \log (1.22)$

Use the log tables:

$D = 10 \times (0.0864)$

$D = 0.864$

Frequently, the antilogarithm must be obtained as shown in Example G.

EXAMPLE G: SOLVING LOGARITHMIC EQUATIONS.

Given: $D = 10 \log (P_2/P_1)$.

$D = 18.7$.

Find: The ratio of P_2/P_1.

Solution: Dividing both sides of the expression by 10, we get:

$$D = 10 \log \left(\frac{P_2}{P_1}\right)$$

$$\log \left(\frac{P_2}{P_1}\right) = \frac{D}{10}$$

Substituting 18.7 for the given value of D, we get:

$$\log \left(\frac{P_2}{P_1}\right) = \frac{18.7}{10}$$

$$\log \left(\frac{P_2}{P_1}\right) = 1.87$$

From the table, we find the antilog of 1.87 to be:

$(P_2/P_1) = 74.1$

Three basic theorems of logarithms that are helpful in solving many problems are:

$\log (xy) = \log x + \log y$
$\log (x/y) = \log x - \log y$
$\log (x^y) = y \log x$

Example H verifies these theorems by numerical examples.

EXAMPLE H: NUMERICAL VERIFICATION OF THEOREMS OF LOGARITHMS.

Given: The three theorems of logarithms stated above, $x = 5$, $y = 2$.

Find: Use these values to verify each theorem.

Solution: a. $\log (xy) = \log x + \log y$

$\log (5)(2) = \log (5) + \log (2)$

$\log (10) = \log (5) + \log (2)$

$1 = 0.699 + 0.301$ (Values obtained from calculation and rounded off)

$1 = 1$

b. $\log \left(\frac{x}{y}\right) = \log x - \log y$

$\log \left(\frac{5}{2}\right) = \log (5) - \log (2)$

$\log (2.5) = \log (5) - \log (2)$

$0.398 = 0.699 - 0.301$

$0.398 = 0.398$

c. $\log (x^y) = y \log x$

$\log (5^2) = 2 \log (5)$

$\log (25) = 2 \log (5)$

$1.398 = (2)(0.699)$

$1.398 = 1.398$

NATURAL LOGARITHMS

Although numbers can be expressed as logarithms (exponents) to any base, only two bases are in general use — (1) the number 10 (common logarithms) and (2) the number e (2.7182818 ....). The symbol "e" comes from the first letter of "exponential" and describes any changing process whose rate of change is proportional to its instantaneous magnitude. Such processes include growth and decay, absorption, capacitor charge, and inductive reactions. The formulas describing these processes contain the symbol e as a base with some exponent. The following are some example equations:

$$q = Q(1-e^{-kt}) \text{ and } y = A\, e^{+kx}$$

Just as any positive number can be written in a power-of-ten form, any positive number can be written in power-of-e form. Since e is approximately 2.72,

$$2.72 = 10^{0.4346}, \text{ or}$$

more accurately,

$$e = 10^{0.4346}$$

The numerical value of any power-of-e can be found by the conversion of power-of-e to power-of-ten form. This conversion simply requires the use of the logarithm table, as shown in Example I.

EXAMPLE I: FINDING e^X.

Given: Table 2, Common Logarithms of Numbers.

Find: e^5.

Solution: $e^5 = (10^{0.4346})^5 = 10^{(0.4346) \times 5}$

$e^5 = 10^{2.1715}$

Use Table 2 of Common Logarithms as shown here:

COMMON LOGARITHMS OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732

THE MANTISSA (1715)

By interpolation: log (149) = 2.1732

log (148.??) = 2.1715

log (148) = 2.1703

$e^5 = 10^{2.1715} = 148 + \left(\frac{0.0012}{0.0029}\right) \times 1$

$= 148 + 0.414$

Therefore, $e^5 = 10^{2.1715} = 148.414$

This method, however, is tedious and time consuming. Mathematicians have prepared a table of power-of-e to three significant digits. Table 3 of this module is a table of e^X or e^{-X} . Locate the power in the x column and read e^X or e^{-X} under the respective column of Table 3:

$e^5 = 148$ (to three significant digits).

TABLE 3. TABLE OF e^x AND e^{-x}.

x	e^x	e^{-x}	x	e^x	e^{-x}	x	e^x	e^{-x}
0.0	1.00	1.00	3.5	33.1	0.0302	7.0	1100	0.000912
0.1	1.11	0.905	3.6	36.6	0.0273	7.1	1210	0.000825
0.2	1.22	0.819	3.7	40.4	0.0247	7.2	1340	0.000747
0.3	1.35	0.741	3.8	44.7	0.0224	7.3	1480	0.000676
0.4	1.49	0.670	3.9	49.4	0.0202	7.4	1640	0.000611
0.5	1.65	0.607	4.0	54.6	0.0183	7.5	1810	0.000553
0.6	1.82	0.549	4.1	60.3	0.0166	7.6	2000	0.000500
0.7	2.01	0.497	4.2	66.7	0.0150	7.7	2210	0.000453
0.8	2.23	0.449	4.3	73.7	0.0136	7.8	2440	0.000410
0.9	2.46	0.407	4.4	81.5	0.0123	7.9	2700	0.000371
1.0	2.72	0.368	4.5	90.0	0.0111	8.0	2980	0.000336
1.1	3.00	0.333	4.6	99.5	0.0101	8.1	3290	0.000304
1.2	3.32	0.301	4.7	110.	0.00910	8.2	3640	0.000275
1.3	3.67	0.273	4.8	122.	0.00823	8.3	4020	0.000248
1.4	4.06	0.247	4.9	134.	0.00745	8.4	4450	0.000225
1.5	4.48	0.223	5.0	148.	0.00674	8.5	4910	0.000204
1.6	4.95	0.202	5.1	164.	0.00610	8.6	5430	0.000184
1.7	5.47	0.183	5.2	181.	0.00552	8.7	6000	0.000167
1.8	6.05	0.165	5.3	200.	0.00499	8.8	6630	0.000151
1.9	6.69	0.150	5.4	221.	0.00452	8.9	7330	0.000136
2.0	7.39	0.135	5.5	245.	0.00409	9.0	8100	0.000123
2.1	8.17	0.122	5.6	270.	0.00370	9.1	8960	0.000112
2.2	9.02	0.111	5.7	299.	0.00335	9.2	9900	0.000101
2.3	9.97	0.100	5.8	330.	0.00303	9.3	10900	0.0000914
2.4	11.0	0.0907	5.9	365.	0.00274	9.4	12100	0.0000827
2.5	12.2	0.0821	6.0	403.	0.00248	9.5	13400	0.0000749
2.6	13.5	0.0743	6.1	446.	0.00224	9.6	14800	0.0000677
2.7	14.9	0.0672	6.2	493.	0.00203	9.7	16300	0.0000613
2.8	16.4	0.0608	6.3	545.	0.00184	9.8	18000	0.0000555
2.9	18.2	0.0550	6.4	602.	0.00166	9.9	19900	0.0000502
3.0	20.1	0.0498	6.5	665.	0.00150	10.0	22000	0.0000454
3.1	22.2	0.0450	6.6	735.	0.00136			
3.2	24.5	0.0408	6.7	812.	0.00123			
3.3	27.1	0.0369	6.8	898.	0.00111			
3.4	30.0	0.0334	6.9	992.	0.00101			

Natural logarithms are based upon the power of "e." The natural logarithm of a number is simply the exponent when it is written in "power-of-e" form. Any power-of-e can be written in logarithmic notation. The symbol "ln" is used when reference is made to logarithms. Notice, for example, these forms:

$$602 = e^{6.4} \quad \text{(power-of-e form)}$$

$$\ln(602) = 6.4 \quad \text{(logarithmic equivalent)}$$

Refer to Figure 2 and compare the form of these equations to the exponential and logarithmic equations shown there. In the case of natural logs the base is "e," and these equations become:

$$y = e^x$$

$$\log_e y = x \quad \text{or} \quad \ln y = x$$

Mathematicians have prepared tables of natural logarithms, and these tables can be found in many handbooks. Table 4 of this module entitled, "Table of Natural Logarithms," is such a representative table. Examine the

TABLE 4. NATURAL LOGARITHMS.

N	ln N	N	ln N	N	ln N	N	ln N
0.00	-------	5.00	1.6094	10.0	2.3026	60.0	4.0943
0.10	-2.3026	5.10	1.6292	11.0	2.3979	61.0	4.1109
0.20	-1.6094	5.20	1.6487	12.0	2.4849	62.0	4.1271
0.30	-1.2040	5.30	1.6677	13.0	2.5650	63.0	4.1431
0.40	-0.9163	5.40	1.6864	14.0	2.6391	64.0	4.1589
0.50	-0.6932	5.50	1.7048	15.0	2.7080	65.0	4.1744
0.60	-0.5108	5.60	1.7228	16.0	2.7726	66.0	4.1896
0.70	-0.3567	5.70	1.7405	17.0	2.8332	67.0	4.2047
0.80	-0.2231	5.80	1.7579	18.0	2.8904	68.0	4.2195
0.90	-0.1054	5.90	1.7750	19.0	2.9444	69.0	4.2341
1.00	0.0000	6.00	1.7[illegible]9	20.0	2.9957	70.0	4.2485
1.10	0.0953	6.10	1.8083	21.0	3.0445	71.0	4.2627
1.20	0.1823	6.20	1.8246	22.0	3.0910	72.0	4.2767
1.30	0.2624	6.30	1.8406	23.0	3.1355	73.0	4.2905
1.40	0.3365	6.40	1.8563	24.0	3.1780	74.0	4.3041
1.50	0.4055	6.50	1.8718	25.0	3.2189	75.0	4.3175
1.60	0.4700	6.60	1.8871	26.0	3.2581	76.0	4.3307
1.70	0.5306	6.70	1.9021	27.0	3.2958	77.0	4.3438
1.80	0.5878	6.80	1.9169	28.0	3.3322	78.0	4.3567
1.90	0.6418	6.90	1.9315	29.0	3.3673	79.0	4.3694
2.00	0.6932	7.00	1.9459	30.0	3.4012	80.0	4.3820
2.10	0.7419	7.10	1.9601	31.0	3.4340	81.0	4.3944
2.20	0.7885	7.20	1.9741	32.0	3.4657	82.0	4.4067
2.30	0.8329	7.30	1.9879	33.0	3.4965	83.0	4.4188
2.40	0.8755	7.40	2.0015	34.0	3.5264	84.0	4.4308
2.50	0.9163	7.50	2.0149	35.0	3.5554	85.0	4.4426
2.60	0.9555	7.60	2.0282	36.0	3.5835	86.0	4.4544
2.70	0.9932	7.70	2.0412	37.0	3.6109	87.0	4.4659
2.80	1.0296	7.80	2.0541	38.0	3.6376	88.0	4.4773
2.90	1.0647	7.90	2.0669	39.0	3.6636	89.0	4.4886
3.00	1.0986	8.00	2.0794	40.0	3.6889	90.0	4.4998
3.10	1.1314	8.10	2.0919	41.0	3.7136	91.0	4.5109
3.20	1.1632	8.20	2.1041	42.0	3.7377	92.0	4.5218
3.30	1.1939	8.30	2.1163	43.0	3.7612	93.0	4.5326
3.40	1.2238	8.40	2.1282	44.0	3.7842	94.0	4.5433
3.50	1.2528	8.50	2.1401	45.0	3.8067	95.0	4.5539
3.60	1.2809	8.60	2.1518	46.0	3.8286	96.0	4.5644
3.70	1.3083	8.70	2.1633	47.0	3.8502	97.0	4.5747
3.80	1.3350	8.80	2.1748	48.0	3.8712	98.0	4.5850
3.90	1.3610	8.90	2.1860	49.0	3.8918	99.0	4.5951
4.00	1.3863	9.00	2.1972	50.0	3.9120	100.	4.6052
4.10	1.4110	9.10	2.2083	51.0	3.9318	150	5.0106
4.20	1.4351	9.20	2.2192	52.0	3.9512	200.	5.2983
4.30	1.4586	9.30	2.2300	53.0	3.9703	250.	5.5215
4.40	1.4816	9.40	2.2407	54.0	3.9890	300.	5.7038
4.50	1.5041	9.50	2.2513	55.0	4.0073	350	4.8579
4.60	1.5261	9.60	2.2618	56.0	4.0254	400.	5.9915
4.70	1.5476	9.70	2.2721	57.0	4.0430	450	6.1092
4.80	1.5686	9.80	2.2824	58.0	4.0604	500.	6.2146
4.90	1.5892	9.90	2.2925	59.0	4.0775		

table, and note that "N" stands for the number and that "ln N" stands for the natural logarithm. Unlike the common log table, the natural log table provides the entire logarithm, as in these examples:

$$\ln (1.60) = 0.4700$$

$$\ln (45.0) = 3.8067$$

The logarithms of numbers lying between two values of N in the table may be determined by interpolation as with common logarithms. Logarithms of numbers above or below the range of the table may be found by application of the theorems of logarithms and the tables as illustrated in Example J.

EXAMPLE J: FINDING NATURAL LOGS EMPLOYING THEOREMS.

Given: Table 4 of Natural Logarithms.

Find: a. $\ln (0.0025)$

b. $\ln (1200)$

Solution: a. Use theorem: $\ln (\frac{x}{y}) = \ln x - \ln y$

Let $x = 1$, $Y = 0.0025$

Solve equation for $\ln y$

$$\ln y = \ln x - \ln (\frac{x}{y})$$

$$= \ln (1) - \ln (\frac{1}{0.0025})$$

$$= \ln (1) - \ln (400)$$

$$= (0) - (5.9915) \quad \text{(from Table 4)}$$

$$\ln (0.0025) = -5.9915$$

b. Use theorem: $\ln (xy) = \ln x + \ln y$

Let $x = 30$, $y = 40$, $(xy = 1200)$

$$\ln (1200) = \ln (30) + \ln (40)$$

$$= (3.4012) + (3.6889) \quad \text{(from Table 4)}$$

$$\ln (1200) = 7.0901$$

Logarithms determined by tables are approximate and are correct to as many significant figures as the number of places in the table used. A table that gives the desired number of significant figures should be used.

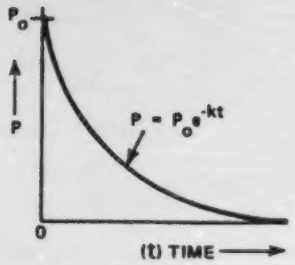

Figure 4. Graph of exponential decay.

APPLICATIONS OF NATURAL LOGARITHMS

A process in which a rate of change of a variable decreases as time proceeds is known as a "decay process." Such a process occurs as an object cools down, as a capacitor discharges, or as radioactive material decays. Such a process is illustrated graphically in Figure 4. The curve is described mathematically by Equation 1.

$$P = P_o e^{-kt} \qquad (1)$$

where: P = A quantity that decays as a function of time.
P_o = Initial value of the decaying quantity.
t = Time.
k = A constant factor controlling the rapidity of the decay (the larger the k value, the more rapid the decay process).

In order to evaluate Equation 1 for the value of P — given $P_o = 40$, $k = 0.02$ and $t = 140$ — substitute the known values as follows:

$$P = P_o e^{-kt}$$
$$= (40)\ e^{-(0.02)(140)}$$
$$= (40)\ e^{-(2.8)}$$

Use Table 3:

$$= 40\ (0.0608)$$
$$P = 2.43$$

Equation 1 can be rearranged to solve for t; for example, given the values $P_o = 40$, $k = 0.02$, and $P = 2.43$, t can be solved thusly:

Step		Explanation
Solution:	$P = P_o e^{-kt}$	Write equation.
	$P e^{kt} = P_o \underbrace{(e^{-kt})(e^{kt})}$	Multiply by e^{+kt}.
	$P e^{kt} = P_o (1) = P_o$	Divide by P.
	$e^{kt} = \frac{P_o}{P}$	
	$kt = \ln (\frac{P_o}{P})$	Write in equivalent logarithmic form.
	$t = \frac{1}{k} \ln (\frac{P_o}{P})$	Divide by k.
	$t = \frac{\ln (40/2.43)}{0.02}$	Substitute.
	$t = \frac{\ln (16.46)}{0.02} = \frac{2.80}{0.02}$	
	$t = 140$	

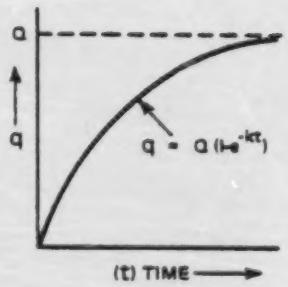

Figure 5. Graph of exponential growth.

Figure 5 illustrates a process that begins with a zero value of a quantity and grows exponentially as time increases toward some maximum value Q. The curve is described mathematically by Equation 2.

$$q = Q (1-e^{-kt}) \qquad (2)$$

Adiabatic expansion of a quantity of an ideal gas is described mathematically by Equation 3.

$$\text{Constant} = \rho V^n \qquad (3)$$

where: ρ = Gas pressure.
V = Initial volume of the gas.
n = Exponent of V.

Logarithms are used, as in Example K, to evaluate Equation 3 for value of the Constant when ρ, V, and n are given.

EXAMPLE K: FINDING THE CONSTANT.

Given: An ideal gas in a container exhibits a pressure of 600 and a volume of 140. The exponent has a value of 1.4. (Units are not considered here, in order that the mathematics itself will be clearer.)

Find: Value of constant.

Solution:

Steps	Explanation
$\text{Constant} = (600)\ (140)^{1.4}$	Susbtitute given data.
$\log (\text{Constant}) = \log [(600)\ (140)^{1.4}]$	Write the logarithmic form.
	Apply a basic theorem of logarithms: $\log (x)\ (y) = \log x + \log y.$
$\log (\text{Constant}) = \log (600) + \log [(140)^{1.4}]$	
	Apply another basic theorem of logarithms: $\log x^n = n \log x.$
$\log (\text{Constant}) = \log (600) + 1.4 \log (140)$	
$= 2.7782 + 1.4 \times 2.1461$	
$= 2.7782 + 3.0045$	
$\log (\text{Constant}) = 5.7827$	Find antilog.
$\text{Constant} = 6.063 \times 10^5$	
or 606300	(To four significant digits.)

LOGARITHMIC GRAPHS

Many of the functions described by logarithmic and exponential functions are commonly represented graphically on logarithmic scale graph paper. Radioactive decay is chosen as a convenient example for illustration.

Figure 6 is a graph on a linear scale showing the fraction of radioactive atoms remaining in a sample after a given elapsed time. The half-life of this particular atom is about 2.2 minutes. That means that one half of the original atoms will have decayed after 2.2 minutes. After an additional 2.2 minutes, half of the remaining atoms will have decayed. Thus, the curve drops rapidly in the beginning and less rapidly as the number of atoms approaches zero. This shape is called an "exponential decay" curve.

Figure 7 shows this same relationship drawn on a semilog graph in which the vertical scale is logarithmic. On this type of graph, the exponential decay curve is a straight line. For this reason, logarithmic graphs are helpful in analyzing exponential functions.

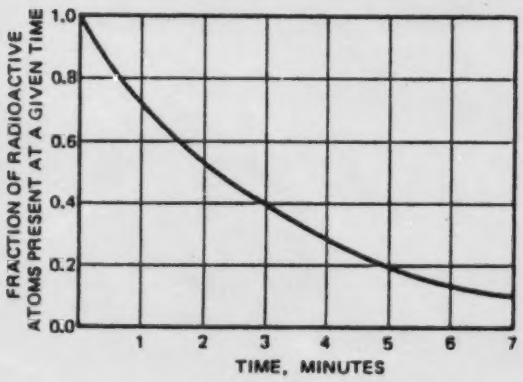

Figure 6. Radioactive decay graph with linear scale.

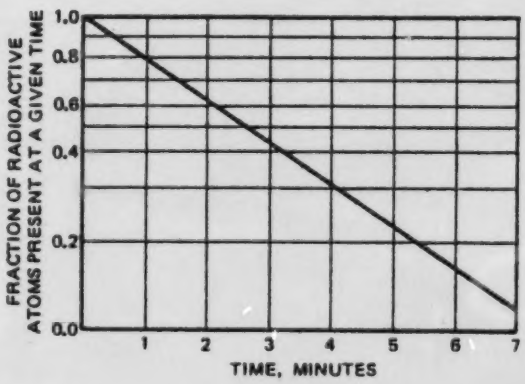

Figure 7. Radioactive decay semilog scale.

The ability to use mathematical equations that include exponential and logarithmic terms is a necessary prerequisite to the study of time constant. Work with logarithms and exponential notation should be done with extreme care. Remember the ease with which one can "trip over logs." On the other hand, properly-handled exponential notation can be as easy as "falling off a log."

SUMMARY

Exponential, or power-of-ten notation, facilitates the recording and manipulation of data by allowing large numbers to be written in abbreviated form.

Logarithms of numbers between 1.00 and 9.99 are called "common" (Table 2). A common logarithm is the exponential expression of a number having ten as the base; that is, the power-of-ten required to produce that number. Logs of numbers greater than 10 are found by conversion to scientific notation and by use of the following rule: "The logarithm of the product of two numbers is equal to the sum of the logarithms of the individual numbers." Often, the antilogarithm, or the number corresponding to a given logarithm, must be found. "Interpolation" is the practice of finding logs of numbers that lie between other numbers. "Natural logarithms" are employed to determine the rate of change in a process; they are valuable in industry.

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REVIEW QUESTIONS

MATERIALS

Logarithm Tables in this module
Scientific calculator

1. Express the following numbers in scientific notation:
   a. 43,762 = _____
   b. 29.78 = _____
   c. 0.013 = _____
   d. 0.000287 = _____

2. Evaluate the following, and express the results in scientific notation. Check your answers with your calculator.
   a. $2.97 \times 10^4 + 3.6 \times 10^1$ = _____
   b. $6.30 \times 10^3 - 6.30 \times 10^2$ = _____
   c. $(1.15 \times 10^{-2}) \times (3.0 \times 10^4)$ = _____
   d. $(4.25 \times 10^6) \div (2.5 \times 10^{-4})$ = _____

3. Determine the logarithms of the following numbers from the tables in this module. Interpolate if necessary. Check your answers with your calculator.
   a. log 235 = _____
   b. log 0.002 = _____
   c. ln 0.065 = _____
   d. ln 16.28 = _____

4. Use the three theorems of logarithms and the tables given in this module to evaluate the following. Check your answers with your calculator.
   a. 18.6×5.2 = _____
   b. $5.0 \div 27$ = _____
   c. $(21.92)^4$ = _____
   d. $(0.025) \times (8)^2 \div (14)^2$ = _____

5. If $D = 10 \log (P_2/P_1)$, find the following:
   a. D when $P_2 = 4$, $P_1 = 3$ D = _____
   b. P_2 when $P_1 = 6$, $D = 20$ P_2 = _____
   c. P_1 when $P_2 = 0.25$, $D = 2.5$ P_1 = _____
   d. D when $P_2 = 1$, $P_1 = 15$ D = _____

6. Express the following numbers in scientific notation (power-of-ten):

   a. 2,930 = _____

   b. 262,000 = _____

   c. 704 = _____

   d. 0.0713 = _____

   e. 0.195 = _____

   f. 0.00000000833 = _____

7. Evaluate the following, and express the results in scientific notation:

   a. $6.61 \times 10^{4} + 5.2 \times 10^{1}$ = _____

   b. $7.36 \times 10^{-6} + 2.74 \times 10^{-4}$ = _____

   c. $6.61 \times 10^{3} - 4.41 \times 10^{4}$ = _____

   d. $(3.4 \times 10^{-3})(1.6 \times 10^{-4})$ = _____

   e. $(4.0 \times 10^{-3})(5.0 \times 10^{3})$ = _____

   f. $\dfrac{6.4 \times 10^{7}}{2.5 \times 10^{-4}}$ = _____

8. Determine the logarithms of the following numbers, using the appropriate tables (interpolate if necessary):

   a. log 505 = _____

   b. log 0.001 = _____

   c. log 9,028 = _____

   d. log 432.3 = _____

   e. ln 0.60 = _____

   f. ln 34.3 = _____

9. Using the three theorems of logarithms and the log tables, evaluate the following:

   a. 21.65×31.3 = _____

   b. $1.0 \div 623$ = _____

   c. $(65.35)^{3}$ = _____

   d. $(65.2)(10.6)^{2} \div 2.314$ = _____

10. In finding the insulation resistance by the leakage method, the resistance R is found from the formula —

$$R = 10^6 \times \frac{t}{c} \times \frac{1}{\ln (V_o V)}$$

where: t = Time, seconds.
c = Capacity.
V = Voltage.

Compute the value of R if t = 130, V_o = 12, V = 115.8, and C = 0.082.

11. In an alternating-current circuit, the current i at any instance is given by the formula:

$$i = I \left[1-e^{-\left(\frac{Rt}{L}\right)}\right]$$

where: i = Instantaneous current.
I = Maximum current.
R = Resistance.
L = Coefficient of self induction.
t = Time.
e = Base of natural logarithm.

Evaluate i when I = 100, R = 16, L = 0.80, and t = 0.02.

12. The voltage in a capacitor discharging is given by the equation —

$$V = E\, e^{-\frac{t}{Rc}}$$

Solve this equation for t, and evaluate the value of t, when E = 200, R = 12, c = 0.05, V = 49.

13. The work done by a volume of gas, expanding at a constant temperature from a volume V_0 to a volume V_1, is given by the following formula:

$$W = p_0 V_0 \ln \left(\frac{V_1}{V_0}\right)$$

Solve this equation for V_1.

14. Select appropriate data points, perform the necessary calculations, construct data tables, and draw on separate sheets of graph paper graphs for the following equations:

a. $y = e^x$ in the range $0 < x \leq 2$

b. $y = \ln x$ in the range $0 < x \leq 20$

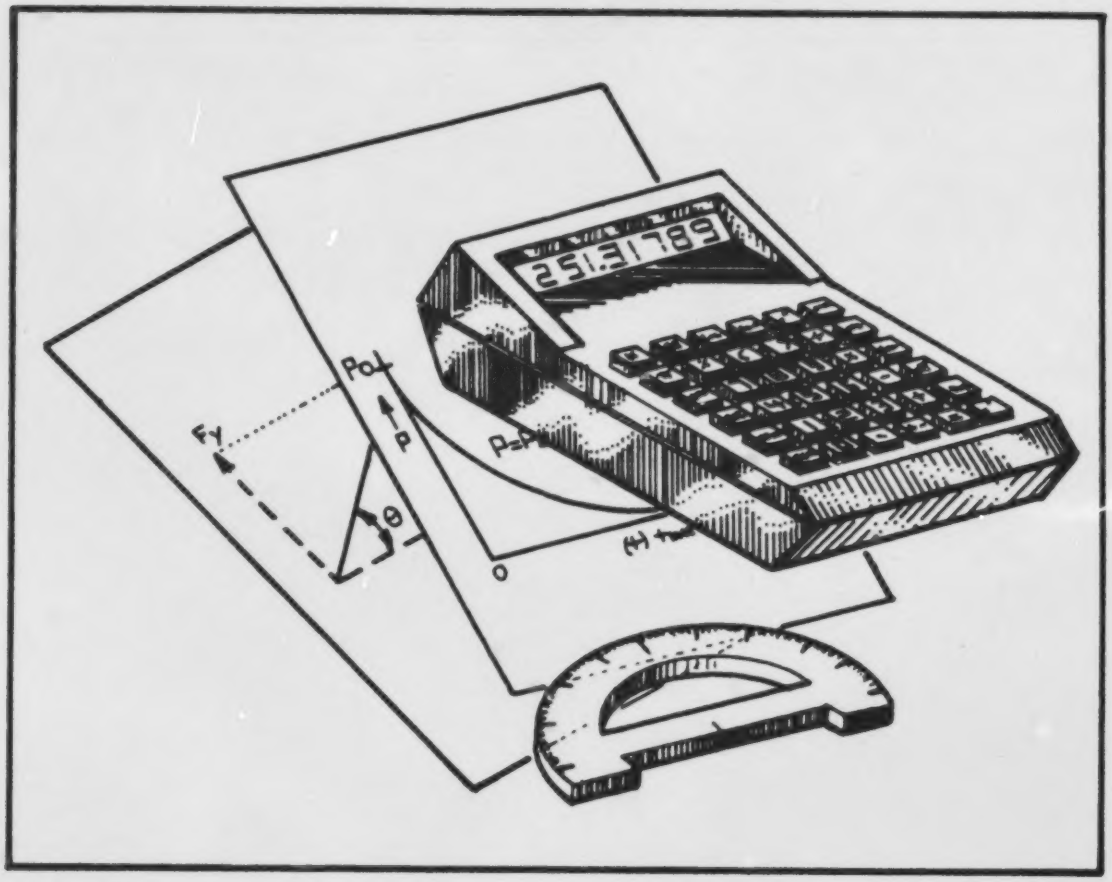
c. $y = e^{-x}$ in the range $0 < x \leq 2$

UTC

UNIFIED TECHNICAL CONCEPTS

MODULE S 8

PRECISION, ACCURACY, AND MEASUREMENT



SUPPLEMENTARY

CORD

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INTRODUCTION

The ability to make precise and accurate measurements is essential to science and technology. Modern life would be impossible without standards of measurement and the knowledge and equipment to apply them. This module discusses the process of measurement. Particular attention is devoted to the accuracy with which measurements can be made and the methods of analyzing and reporting this accuracy.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Distinguish between the following pairs:
   a. Counting and measurement.
   b. Precision and accuracy.
   c. Schematic and random errors.
   d. Direct and indirect measurements.
2. Given a set of data from an instrument with a known limit of error, calculate the limit of error for the resulting measurement.
3. Given a measurement and limit of error, write an expression with the proper significant figures that indicated the limit of error.
4. Calculate the result and the limit of error when measurements with a known limit of error are added, subtracted, multiplied, or divided.
5. Measure linear dimensions with a vernier caliper to the nearest 0.1 mm and with a micrometer caliper to the nearest 0.001 mm.

COUNTING AND MEASUREMENT

Two distinct methods exist for determining quantities. Counting (Figure 1) allows the exact number of distinct objects to be determined. This process yields an answer in the form of an integer and involves no error if done carefully. Measurement is used to determine an unknown quantity that cannot be counted by comparison to a standard. Four types of measuring instruments are shown in Figure 2 with the quantities they are designed to measure. Even with the most accurate instruments and the greatest care, measuring always results in some error.

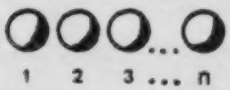

Figure 1. An integral number of objects.

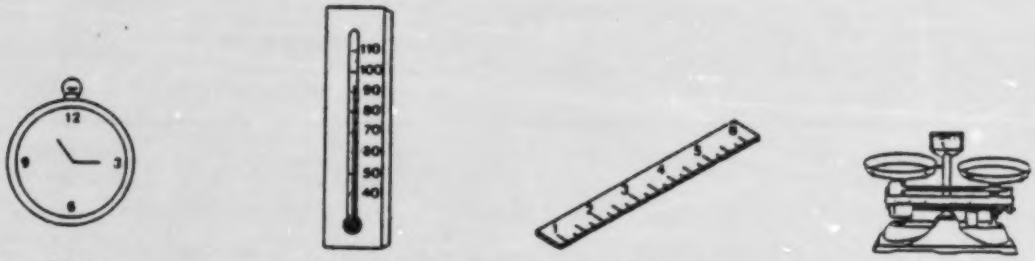

Figure 2. Measurement of four basic quantities.

An expression of a measured quantity always must contain a reference to the standard used in the measurement. An expression such as "the length of the string is 25" has no meaning. It could be 25 miles or 25 millimeters. There are three characteristics of any good standard (or unit) of measurement:

- It must be a constant. Measurement of distances in paces is not acceptable because a "pace" is different for different people.
- It must be easily reproduced and accessible. Measuring distances in terms of the wavelength of light is very accurate, but the standard unit and techniques are not available to most people.
- It must be understood and agreed to by everyone involved. For example, lack of understanding is a problem that must be overcome in the United States in converting to the Metric system.

"Precision" refers to the repeatability of a measurement. If a measurement of the same quantity is repeated several times with the same instrument and with perfect precision, the result will be the same each time. Accuracy refers to the agreement of the measured value with the actual or true value. These concepts are illustrated in Table 1.

TABLE 1. LENGTH OF 25 cm ROD AS MEASURED BY THREE STUDENTS.

Trial	Student 1	Student 2	Student 3
1	24.8 cm	26.2 cm	25.0 cm
2	22.5 cm	26.1 cm	25.1 cm
3	27.8 cm	26.2 cm	24.9 cm
4	26.0 cm	26.3 cm	25.1 cm
5	23.5 cm	26.2 cm	25.0 cm
Evaluation	Not precise or accurate	Precise but not accurate	Precise and accurate

Three students referred to in Table 1 have made five measurements each of the length of a rod having a true length of 25 cm. The values obtained by student 1 vary greatly; therefore, these measurements are not precise, and if precision is lacking, accuracy is in doubt. Student 2 has achieved greater precision, but has obtained results that are not accurate. Precision does not assure accuracy. Student 3 has achieved both accuracy and precision in his/her measurements.

The accuracy and precision of measurements are determined by three classes of errors. Systematic errors usually are determinable and are due to assigned causes. They affect the accuracy of a measurement, but not the precision. Examples are a speedometer that consistently indicates a higher speed than the true value, a warped meter stick, and a constant parallax error in reading a voltmeter. Random errors usually are erratic and unpredictable and cannot always be assigned to known causes. They affect the precision of a measurement, but not necessarily the accuracy. Examples of random errors are power surges in electronic measuring equipment, changes in

parallax in reading scales, and inability to repeat any measurement exactly the same way twice. Both systematic and random errors are always present in any measurement. Reliable results require that the errors be as small as practical and that the amount of error be known and specified. The third class of errors is called gross errors and are the result of mistakes on the part of the experimenter. Gross errors may be eliminated by an experienced and knowledgeable experimenter. Figure 3 is a chart showing the sources of all three types of errors.

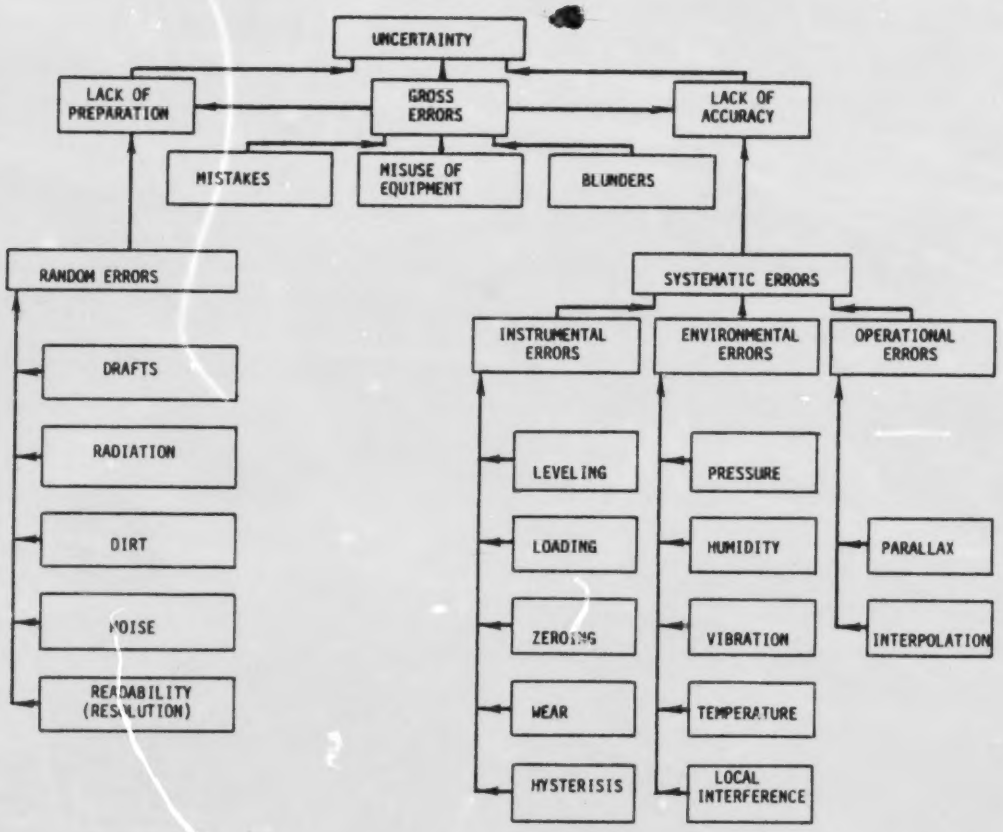

Figure 3. Classification of measurement errors.

SPECIFYING ERRORS IN MEASUREMENT

The instrument limit of error (I.L.E.) is the systematic error built into the measuring device. The magnitude of this error depends upon the manufacturing process used. The instrument limits of error for several common-length measurement devices are given in Table 2. If no limit of error is specified for a device, it generally is assumed to be one-half the smallest scale division. Electronic meters normally are assumed to have an accuracy of 2% of the full-scale reading unless otherwise specified.

TABLE 2. ERRORS OF INSTRUMENTS FOR MEASURING LENGTH.

Measuring Instrument	Instrument Limit of Error
Wooden meter stick	0.4 mm
Plastic scale (linear)	0.2 mm
Metal scale (linear)	0.1 mm
Vernier caliper	0.05 mm
Micrometer caliper	0.005 mm

Random errors are analyzed from calculation of the statistical limit of error (S.L.E.) from Equation 1.

$$\text{Statistical limit of error} = \frac{4 \times \text{average deviation}}{\sqrt{\text{number of measurements}}}$$

$$\text{In Symbols:} \qquad \text{S.L.E.} = \frac{4\ \text{a.d.}}{\sqrt{n}} \tag{1}$$

The use of this equation is illustrated by Example A.

EXAMPLE A: CALCULATION OF STATISTICAL LIMIT OF ERROR.

Given: Nine readings of length are listed in the below table.

Find: Length and statistical limit of error.

Solution:

	Readings (cm)	Deviations (cm)
	96.44	0.13
	96.12	0.19
	96.28	0.03
	96.34	0.03
	96.45	0.14
	96.38	0.07
	96.11	0.20
	96.38	0.07
	96.30	0.01
Total	866.80 cm	0.87 cm
Average	96.31 cm	0.097 cm

Compute average deviation (a.d.):

1. Total all readings.
2. Divide total by number of readings to find average (96.31 cm).
3. Find difference between each reading and the average. List all deviations as positive quantities; for example —

   (96.44 - 96.31 = 0.13)

   (96.31 - 96.12 = 0.19)

4. Total deviations.
5. Divide total deviation by number of readings for average deviation.

$$S.L.E. = \frac{4\ a.d.}{\sqrt{n}}$$

$$= \frac{(4)\ (0.097\ cm)}{\sqrt{9}}$$

$$= \frac{0.388\ cm}{3}$$

$$S.L.E. = 0.13\ cm$$

Result: Length = 96.31 ± 0.13 cm

The result obtained in Example A indicates that there is a good chance (something like 600:1 odds) that the true length of the object being measured is between 96.18 and 96.44 cm (written as 96.31 ± 0.13 cm) if there is no error in the measuring device itself (which never is the case). The limit of error (L.E.) for the measurement must include both the statistical limit of error and the instrument of error, as shown in Equation 2.

$$\text{Limit of Error} = \text{Statistical limit of error} + \text{Instrument limit of error}$$

$$\text{In Symbols:}\quad L.E. = S.L.E. + I.L.E. \qquad (2)$$

The use of this equation is shown in Example B.

EXAMPLE B: CALCULATION OF LIMIT OF ERROR.

Given: A length of 96.31 ± 0.13 cm is measured.
I.L.E. = 0.4 mm (wooden meter stick, Table 2).

Find: True length and limit of error.

Solution:

$$L.E. = S.L.E. + I.L.E.$$

$$= 0.13\ cm + 0.04\ cm$$

$$L.E. = 0.17\ cm$$

Length = 96.31 ± 0.17 cm

The discussion thus far has considered only "direct" measurements — those in which some quantity is compared to a standard. "Indirect" measurements are the results of calculations involving one or more quantities determined by direct measurements. The following rules apply in determining the limit of error of indirect measurements:

- ADDITION

  Suppose $C = A + B$, where $A = 200.1 \pm 0.2$ cm (direct measurement).
  $B = 10.2 \pm 0.1$ cm (direct measurement).
  Then $C = 210.3 \pm 0.3$ cm (indirect measurement).

  The rule in addition simply is to add the limits of error; i.e., $(\pm 0.2 \text{ cm}) + (\pm 0.1 \text{ cm}) = \pm 0.3$ cm.

- SUBTRACTION

  Suppose $C = A - B$, where $A = 200.1 \pm 0.2$ s
  $B = 10.2 \pm 0.1$ s
  Then $C = 189.9 \pm 0.3$ s

  The rule in subtraction is the same as for addition. Although the mean values are subtracted (200.1 s - 10.2 s), the limits of error (± 0.2 s and ± 0.1 s) are added.

- MULTIPLICATION

  Suppose $C = A \times B$, where $A = 200 \pm 2$ m, or $A = 200$ m $\pm 1\%$, since $2/200 = 1\%$.
  $B = 10 \pm 0.4$ m, or $B = 10$ m $\pm 4\%$, since $0.4/10 = 4\%$.
  Then $C = (200 \text{ m} \times 10 \text{ m}) \pm 1\% \pm 4\%$
  $= 2000 \text{ m}^2 \pm 1\% \pm 4\%$
  $= 2000 \text{ m}^2 \pm 5\%$

  Notice that the limits of error for both measurements A and B first were converted to a relative or percentage limit of error. That is, $A = 200 \pm 2$ m (where 200 m is the mean value and ± 2 m is the limit of error) was written as 200 m $\pm 1\%$. Similarly, B became 10 m $\pm 4\%$. C was obtained by multiplying the mean values directly to obtain 2000 m^2 and adding the relative or percentage errors to obtain $\pm 5\%$. Thus, C was given as 2000 $m^2 \pm 5\%$. Now 5% is a relative

error. If C is to be written in terms of an absolute limit of error, it becomes C = 2000 m^2 ± 5% of 2000 m^2, or C = 2000 ± 100 m^2. Thus the rule for propagation of errors in multiplication is to add the relative or percentage errors associated with each direct measurement.

- DIVISION

  Suppose C = A/B, where A = 200 ± 2 m = 200 m ± 1%

  B = 10 ± 0.4 s = 10 s ± 4%

  Then C = 200 m/10 s ± 5% = 20 m/s ± 5% = 20 ± 1 m/s

  The rule is the same as that for multiplication. The errors first are converted to relative or percentage errors and then added to obtain the overall percentage error associated with the quotient.

- POWER AND ROOTS

  Since a number raised to a given power, such as $C = A^3$, is an example of multiplication, the relative error for C would be three times the relative error for A. Likewise, for a number such as $C = A^{1/3}$, the relative error for C would be 1/3 times the relative error for A.

In reporting the results of a measurement, the numerical value must be expressed with the proper number of significant figures. In Example A, a meter stick was used to measure a length. The smallest division on a meter stick is 0.1 cm, and all readings were estimated to the nearest tenth of that division, or 0.01 cm. Calculations with a pocket calculator give a result of 96.311111 ± 0.1288889 cm. Writing all these digits indicates greater accuracy than actually was achieved. The final answer is always rounded off to leave the same number of digits to the right of the decimal as the original readings, in this case 96.31 ± 0.13 cm. All calculations preferably should be completed before the answer is rounded off, to avoid the introduction of additional errors.

When indirect measurements are calculated from two or more direct measurements with different significant figures, the answer should be expressed with the same number of significant figures as the direct measurement with the lowest number of significant figures, as indicated in the velocity calculation of Example C.

EXAMPLE C: VELOCITY CALCULATION.

Given: Velocity = $\frac{\text{distance}}{\text{time}} = \frac{d}{t}$

distance: $d = 85.15 \pm 0.12$ cm

time: $t = 22 \pm 1$ s

Find: Velocity and limit of error.

Solution: $d = 85.15 \pm 0.12$ cm $= 85.15 \pm 0.14\%$

$t = 22 \pm 1$ s $= \pm 4.5\%$

$v = \frac{d}{t}$

$= \frac{85.15 \text{ cm}}{22 \text{ s}} \pm (0.14\% + 4.5\%)$

$v = 3.87045$ cm/s $\pm$ (4.64%)

$v = 3.9$ cm/s $\pm$ 4.6% (Round off to 2 significant figures.)

$v = 3.9 \pm 0.2$ cm/s

THE READING OF SCALES

When reading linear scales, the smallest unit to which a measurement can be estimated is one-tenth the smallest scale division. For scales with small divisions, the practical limit may be only one half of a division.

Figure 4. Vernier caliper.

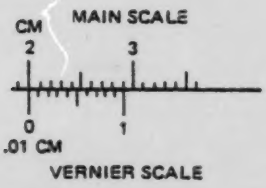

Figure 5. Scales on vernier caliper.

Figure 4 shows a vernier caliper that affords greater accuracy. This device incorporates two scales, as indicated in Figure 5. The main scale has divisions at a spacing of 0.1 cm. The vernier scale is 10% smaller. When a length is measured, the vernier scale slides along beside the fixed scale. The first two significant figures of the length are determined by the position of the zero mark on the vernier scale; the third is the vernier scale mark that lines up with any mark on the main scale. The length in Figure 5 is 2.00 cm. Figure 6 is another example.

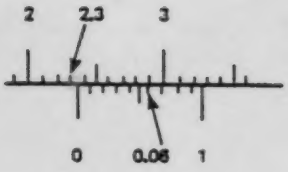

Figure 6. Another vernier scale.

Figure 7. Micrometer caliper.

The zero mark of the vernier scale is just past the 2.3 mark on the main scale, and the 0.06 mark of vernier scale lines up with a mark on the main scale. The measured length is 2.36 cm.

The micrometer caliper in Figure 7 is still more accurate. Figure 8 displays the scales of a micrometer calibrated in ten thousandths of an inch. The smallest main-scale divisions are 0.025 in, the thimble-scale divisions are 0.001 in, and the vernier scale divisions are 0.0001 in. The reading in Figure 8 is 0.2500.

Figure 9 is another example of a micrometer reading. The components of the indicated length are as follows:

main scale –	0.1750 in
thimble scale –	0.0110 in
vernier scale –	0.0004 in
Total	0.1864 in

Some micrometers lack the vernier scale. These are read to the nearest thousandths of an inch (or 0.001 cm for metric micrometers) on the thimble scale. The next digit may be estimated if desired.

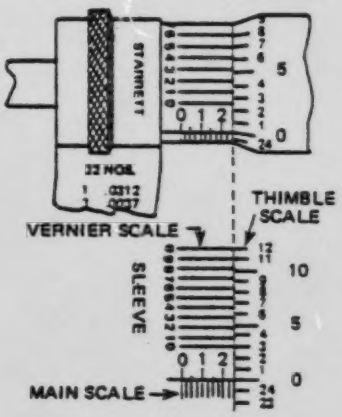

Figure 8. Scale of micrometer.

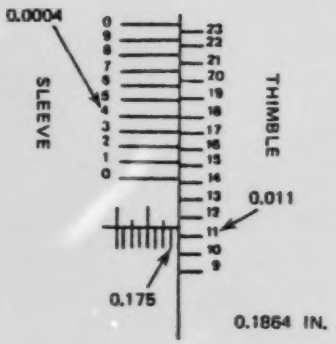

Figure 9. Micrometer reading.

SUMMARY

In the technological world of today, precision measurement is a strict requirement. "Precision" refers to the repeatability of a measurement; "accuracy," to the agreement of the measured value with the actual, or true, value.

Systematic errors in measurement result from the imperfections of equipment; whereas, random errors result from imperfect observation or technique. The instrument limit of error (I.L.E.) is the systematic error built into the measuring device. Significant figures must be used to express a numerical value with accuracy.

The use of the vernier caliper, of the micrometer, and of the variety of other scales and devices is important and must be understood by the technician.

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MATERIALS

Wooden meter stick
Vernier caliper (metric)
Micrometer caliper (metric)
Metal cylinder, approximately 1.20 cm (1/2 inch) in diameter and 1.90 cm (3/4 inch) in length

1. A meter stick was used in making 9 diameter and length measurements of a cylinder. The measurements, made to the nearest 0.01 cm, are recorded in the following Data Table 1.

DATA TABLE 1.				
METER STICK I.L.E. =				
Trial	Length (cm)	Deviation (cm)	Diameter (cm)	Deviation (cm)
1	22.15		6.15	
2	22.18		6.09	
3	21.96		5.98	
4	21.57		5.96	
5	22.00		6.01	
6	22.13		6.06	
7	21.87		5.99	
8	22.05		6.11	
9	22.17		6.03	
Totals				
Average				

a. From the recorded data, calculate the deviation (in centimeters) for each reading and record the results of your calculations in the above Data Table 1.
b. Locate the instrument limit of error (I.L.E.) in Table 2 (page 6 of Subject Matter) and record the value on Data Table 1 above.

c. Complete the calculations required in the following Data Table 2 to determine the volume of the cylinder and limit of error of volume.

2. Complete calculations in Data Table 2 to determine volume of cylinder and limit of error of volume. (Refer to Example A.)
3. Use the vernier caliper and then the micrometer caliper to make 9 measurements of the diameter of the cylinder. Read each instrument for the greatest possible accuracy. Record values in Data Table 3. (Refer to Figure 8).
4. In Data Table 3, record the instrument limits of error from Table 2 of Discussion.
5. Calculate the diameter and limit of error for each set of data in Data Table 3. Record results in Data Table 4.
6. Compare values obtained for diameter and limit of error with the three measuring devices. If different values are obtained with the three devices, explain why.

DATA TABLE 2.

LENGTH

Statistical Limit of error: $S.L.E. = \frac{4\ a.d.}{\sqrt{n}}$

$= (4)\ (\_\_\_\_\ cm) \div \sqrt{9}$

$= \_\_\_\_\_\ cm \div 3$

$S.L.E. = \_\_\_\_\_\ cm$

Limit of error: $L.E. = S.L.E. + I.L.E.$

$= \_\_\_\_\ cm + \_\_\_\_\ cm$

$= \_\_\_\_\ cm$

Measured length: $l = \_\_\_\_\ \pm \_\_\_\_\ cm$

DATA TABLE 2. CONTINUED.

DIAMETER

Statistical limit of error: S.L.E. = _____ cm

Limit of error: L.E. = _____ cm

Measured diameter: d = _____ ± _____ cm

VOLUME

Mean volume: $V = \frac{\pi d^2 l}{4}$

$= (\frac{3.14}{4})\ (\text{———})^2\ (\text{———})$

V = _____ cm^3

Limit of error of volume:

% error in diameter = $\frac{\text{L.E. of diameter}}{\text{mean diameter}}$ (100) = _____%

% error in length = $\frac{\text{L.E. of length}}{\text{mean length}}$ (100) = _____%

% error in volume = (2 x % error in diameter) + error in length

= _____%

Limit of error in volume: L.E. = mean volume x % error in volume ÷ 100

L.E. = _____ cm^3 x _____ ÷ 100

L.E. = _____ cm^3

Volume = _____ ± _____ cm^3

DATA TABLE 3.								
TRIAL NO.	VERNIER CALIPER I.L.E. =				MICROMETER CALIPER I.L.E. =			
	L in CM	Dev. in CM	D in CM	Dev. in CM	L in CM	Dev. in CM	D in CM	Dev. in CM
1								
2								
3								
4								
5								
6								
7								
8								
9								
Total								
Avg.								

DATA TABLE 4.	
VERNIER CALIPER	MICROMETER CALIPER
LENGTH (L)	
S.L.E. = (4)(a.d.)[1/(n)] cm	
S.L.E. = (4)(___)[1/(√___)]	S.L.E. = (4)(___)[1/(√___)]
S.L.E. = _____ cm	S.L.E. = _____ cm
L.E. = S.L.E. + I.L.E.	
L.E. = _____ cm + _____ cm	L.E. = _____ cm + _____ cm
L.E. = _____ cm	L.E. = _____ cm
Measured Length: L = average length in cm ± L.E. in cm	
L = _____ cm ± _____ cm	L = _____ cm ± _____ cm

DIAMETER (D)

S.L.E. = (4)(___)[1/(√___)]

S.L.E. = _____ cm

L.E. = _____ cm + _____ cm

L.E. = _____ cm

Measured Diameter: D = average diameter in cm ± L.E. in cm

D = _____ cm ± _____ cm

S.L.E. = (4)(___)[1/(√___)]

S.L.E. = _____ cm

L.E. = _____ cm + _____ cm

L.E. = _____ cm

D = _____ cm ± _____ cm

VOLUME (V) Mean Volume $V = (\frac{1}{4})D^2L$

$V = (\frac{1}{4})(\ )(\_\_\_\_\_)^2(\_\_\_\_\_)$

$= (0.7854)(\_\_\_\_\_)(\_\_\_\_\_)$

V = _____ cm^3

$V = (\frac{1}{4})(\ )(\_\_\_\_\_)^2(\_\_\_\_\_)$

$= (0.7854)(\_\_\_\_\_)(\_\_\_\_\_)$

V = _____ cm^3

Limit of Error in Volume

$\%\ \text{error in D} = \frac{\text{L.E. of D}}{\text{mean D}}(100)$

$\%\ \text{error D} = \frac{(\quad)}{(\quad)}(100)$

% error D = _____%

$\%\ \text{error D} = \frac{(\quad)}{(\quad)}(100)$

% error D = _____%

$\%\ \text{error in L} = \frac{\text{L.E. of L}}{\text{mean L}}(100)$

$\%\ \text{error L} = \frac{(\quad)}{(\quad)}(100)$

% error L = _____%

$\%\ \text{error L} = \frac{(\quad)}{(\quad)}(100)$

% error L = _____%

% error in Volume = (2)(% error D) + (% error L)

% error V = (2)(_____) + (_____)

% error V = _____%

% error V = (2)(_____) + (_____)

% error V = _____%

L.E. = (mean V)[(% error V)(1/100)]

$L.E. = (\_\_\_\_\_)(\frac{\quad}{100})$

L.E. = _____ cm^3

$L.E. = (\_\_\_\_\_)(\frac{\quad}{100})$

L.E. = _____ cm^3

Calculated Volume of Cylinder

V = _____ cm^3 ± _____ cm^3

V = _____ cm^3 ± _____ cm^3

8. A metal scale was used to make five sets of measurements for determining the volume of a rectangular solid. The measured values were:

Trial	Length (mm)	Width (mm)	Height (mm)
1	22.5	15.3	6.4
2	22.4	15.2	6.3
3	22.5	15.4	6.2
4	22.5	15.4	6.2
5	22.3	15.2	6.1

On a separate sheet of paper, complete an error analysis of these measurements and the resulting volume. Your analysis should be similar to that in Question 1. In this case, the volume is: $V = lwh$.

9. Name two quantities commonly counted and two that are measured. State the difference between precision and accuracy.

10. Name one systematic error and one random error possibly associated with Questions 1 and 8.

11. The following measurements were made of the diameter of a ball bearing with a vernier caliper. Determine the average diameter and limit of error.

Trial	Diameter (cm)
1	1.27
2	1.26
3	1.27
4	1.28
5	1.28
6	1.29
7	1.27
8	1.29
9	1.28

I.L.E. from Table 2

12. The volume of a sphere is given by $V = (4/3)\pi r^3$ or $4\pi r^3/3$. Calculate the volume of the ball bearing above and the limit of error of the volume.

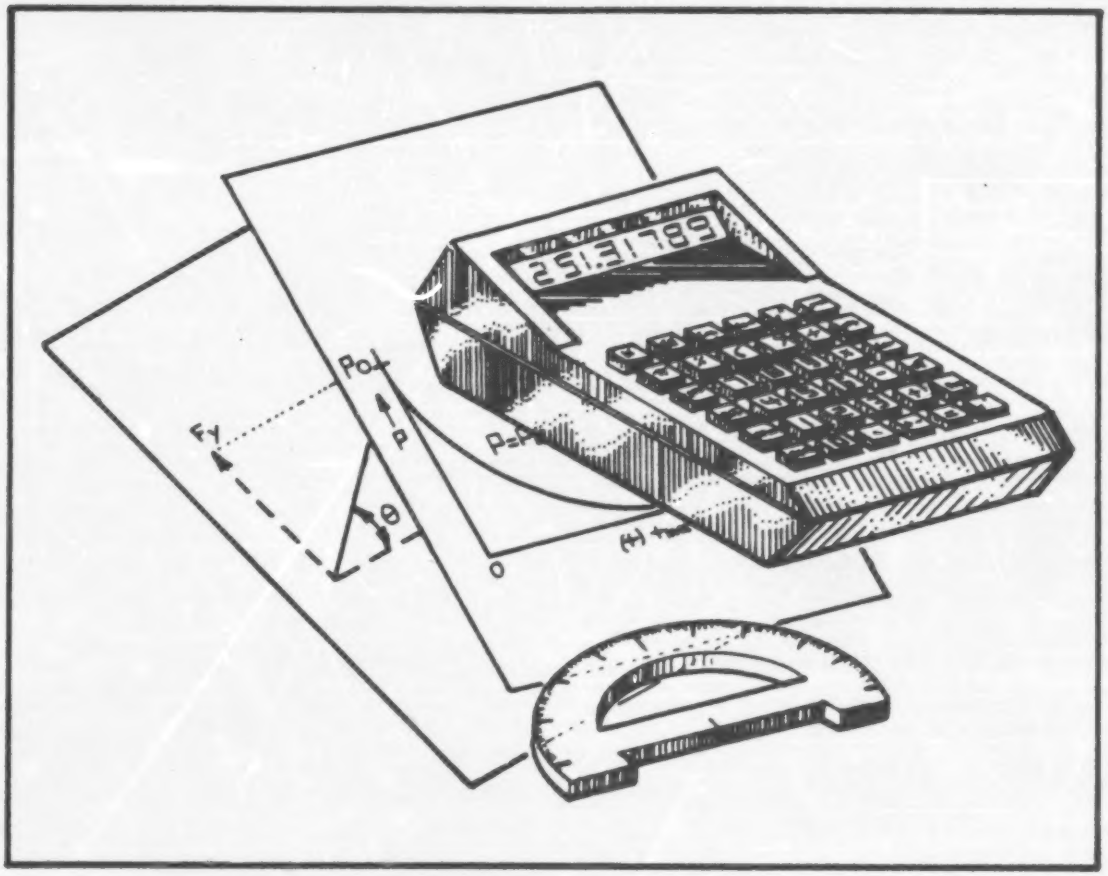
13. Explain the difference between counting and measuring.

UTC

UNIFIED TECHNICAL CONCEPTS

MODULE S 9

LABORATORY NOTEBOOK PREPARATION



SUPPLEMENTARY

CORD

CENTER FOR OCCUPATIONAL RESEARCH AND DEVELOPMENT

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INTRODUCTION

By definition, a logbook or notebook is classified as a permanent record of sequential data and notes about the ideas, progress, results, or performance of or about something. A specific example would be recording the concept, design, construction, and test results of new computer circuit.

The technician or technologist will usually be involved with two types of written records:

1. The technical or progress report.
2. The laboratory logbook.

Both records contain detailed technical information with results, conclusions, recommendations, and so on, but they are not prepared in the same format or at the same time. The technical report is a more formal report, containing technical drawings, illustrations, photographs, line drawings, and so on. This type of report usually is edited before distribution to various key individuals or to the public. The technical or progress report is prepared with the aid of the laboratory logbook at the end of a specific time or at the termination of a project. An example of this would be called a quarterly progress report or a final report. These types of reports are usually the responsibility of an immediate supervisor or project engineer. The laboratory logbook, however, is a continuing record of daily activities and is prepared in chronological order as experimental work actually is being performed.

Suggestions and examples provided in this module will help the student to prepare a laboratory logbook. The student must remember that keeping a laboratory logbook is part of the technician's responsibility, and that preparing this record becomes easier with practice if guidelines are followed.

OBJECTIVES

Upon completion of this module, the student should be able to demonstrate a knowledge of laboratory notebook preparation by making statements, definitions, descriptions, and illustrations to the following:

1. Describe a systematic method of maintaining a laboratory logbook.
2. Explain working habits that a student must develop for performance of experimental work in which the recording of data is mandatory.
3. Define the methods used to present experimental data in graphical form.
4. Describe the recording of experimental data as described in this module.
5. Illustrate, by example, the technique of preparing a laboratory logbook.

SUBJECT MATTER

The need for laboratory logbooks can be broken down into two broad categories. One, there is a reasonable expectation that patentable material will result from the laboratory work. Two, the main purpose is retrieval of information for a report or to find out if an experiment can be repeated. Either of these types might be used in defense of a liability suit.

In both types, it is important that all of the data be original data, that the data is recorded, that any background necessary to the understanding of the experiment be written, that the conclusions are written, and the book is signed and witnessed.

LOGBOOKS FOR POTENTIALLY PATENTABLE EXPERIMENTS

In this type, the primary purpose of the laboratory logbook or notebook is to protect personal and corporate patent rights and other legal rights by providing records of all original work in a form acceptable as evidence in a court of law if any legal conflict should arise. The technician must remember that data copied from a piece of scratch paper, or the back of an envelope for subsequent transfer to the logbook does not serve as official data. This procedure does not follow the primary purpose of the logbook. Consequently, the technician must learn to record raw data, experimental concepts, diagrams, layouts, plans, procedures, and discussions with suppliers and others outside the company in a form that may be read and interpreted as the experiment is performed. Furthermore, this procedure prevents the student from making the laboratory logbook a work of art developed outside the laboratory.

The method described here for keeping a laboratory logbook is certainly not exclusive; there are many variations and the technician will most likely develop his or her own style later. The important point is to be aware of what is to be included in the logbook and that the reader of the logbook should have no doubt about who recorded the data during the experiment or when, why, how, and where the experiment was performed and the data was collected.

NAME, DATE, TIME

To conform to patent laws, the technician's full name, along with the date and the time of day, must appear at the beginning of each day's work. Some logbooks used in research and development organizations require that a technician also sign the logbook at the end of the day and have it countersigned and dated by two witnesses who are not co-inventors. Record the names of operators and other witnesses present during any demonstrations of an important experiment.

INTRODUCTION, OBJECTIVES

Include a statement that will explain why the experiment is to be performed. Include any spur-of-the-moment ideas that may be followed up at a later date. Explain in the introduction what you intend to do or study.

REFERENCES

List the title of any book, magazine article, technical note, or conversations with other people related to the experiment.

Example: Reference Data for Radio Engineers. 5th ed., pp 22-41, Fig. 40 - Attenuation of Cables. Discussion with William Collins, E G & G Co., Boston, MA.

This item may not be needed in all cases.

INSTRUMENTS, COMPONENTS AND SUPPLIES

At times, the technician may be asked to repeat the experiment to determine why unusual results have been obtained. These results could be due to a faulty instrument, which the technician can prove only by identifying which instrument was used in obtaining the measurement. Many times an engineer will not inform the technician what to do except from the measurement and, upon analysis of the measurement, the engineer will compare the measured results to the theoretical calculations. If a discrepancy exists, the engineer may ask the technician to verify the measurements by repeating the experiment. It may be quite embarrassing to the technician to repeat the operation

with a different instrument because he or she could not identify the instrument used. If there is any chance of this, the technician should record the necessary identification (manufacturer and/or serial numbers).

LAYOUTS, DIAGRAMS, SCHEMATICS

Include a sketch of the experiment. A good starting point is the use of a block diagram. The block diagram can be followed by a wiring diagram or schematic drawing. A drawing template saves time and gives a neat appearance to drawings. Leave plenty of room for changes at a later date if necessary. Label all drawings or any sketches with a figure number and a title. The purpose of this notation is to provide an easy reference in discussion and conclusion following the end of each day's work or the end of the experiment.

PHOTOGRAPHS, PHOTOCOPIES, CLIPPINGS

Use long-lasting glue or tape to attach photographs, photocopies, and loose articles of information to each page of the logbook. Years ago, rubber cement was used extensively to paste photographs and sheets of graph paper to the logbook, but this practice is not recommended because the glue deteriorates with age and the data can be easily lost. Regular Scotch brand tape will also become useless after a period of three to four years. A newer type of tape called "Scotch Magic Tape" is recommended. It has a satin finish which is easy to write on with pen or pencil and will not deteriorate with age. The use of staples is also recommended to secure articles.

CALCULATIONS

If you are using a bound, journal-type logbook, a good practice is to use the left side of the page for calculations so they will be available if you want to recheck them. It is poor practice to make calculations on a separate piece of paper, enter the results into the logbook, and then discard the piece of paper with all the calculations on it. It is not really necessary to show repeated simple calculations unless one desires this. Enter any calculations systematically in a neat and orderly manner so that someone can interpret them reasonably well.

TABULATED DATA

In almost all experiments a number of measured values must be recorded. Tables or charts are the most convenient method for recording this type of data, as all values are readily available for checking or analysis. Be sure to include a title at the top of the table and assign a table number to the data shown. It is important that proper units be included in the column headings.

PROCEDURE

When starting a page, enter project title, project number, and book register number. Always use ink for permanence. Record work as you progress, including any spur-of-the-moment ideas that may be developed at a later date. Short comments near recorded data charts about measurements, instruments used, and special techniques are usually sufficient to explain the procedure. Record work in such a manner that a co-worker can continue from where you stop. You might be ill or transferred to another project, and to protect your priority it could be urgent that the work continue during your absence. Give a complete account of your experiments and results, both positive and negative, including your observations.

ERRORS AND CORRECTIONS

Do not try to erase incorrect entries; draw lines deleting them, note the corrections, sign and date the changes. It should be understood that all erasures are observed with suspicion. This extra care is worthwhile because of the need for original data to prove the priority of new discoveries. In some experiments the possible error is the only interesting part of the experiment and the measurement would be completely lost if it were completely erased.

GRAPHS

Pages of most laboratory logbooks are printed with quadrilleruled line technique. Cross-ruled lines, blue in color, are spaced in divisions of either four or five to the inch. This paper has the same appearance as square-

grid graph paper and is very useful for drawings, graphs or tables of data. However, it does not provide the accuracy required under some circumstances. It may be necessary to plot data on a sheet of 8 1/2 x 11 inch graph paper and attach the graph paper to the logbook as previously described. A graph is perhaps one of the most effective means of communication that any technically trained person can use. Essentially, a graph is a pictorial representation of data collected and tabulated throughout an experiment. Because this information must be presented honestly, accurately, and clearly, skill and good judgment are required to make an attractive graph. Some of the most difficult problems in constructing a graph are choosing the correct type of graph paper - rectilinear, logarithmic, semilogarithmic, polar - and then deciding upon proper vertical and horizontal scales to use so that data may be easily plotted and easily read.

Data to be plotted are generally available in tabular form, with each point having two coordinates - x and y. Standard practice is to plot the independent variable along the horizontal axis and the dependent variable along the vertical axis. The independent variable may be controlled during a test or measurement.

After a graph has been laid out and the points have been plotted, the problem of drawing or fitting a curve to the plotted points is presented. The manner in which the data points are connected together depends upon several factors:

1. If data is continuous, a smooth, curved line drawn with the aid of an irregular or French curve, is made through or near the plotted points.
2. If data are discontinuous (i.e., data representing a discontinuous variable having discrete elements), straight lines should be drawn between successive data points, or a bar chart may be preferable.
3. Data that represents a continuous function should be plotted and then a smooth curve should be drawn through and as near the points as possible. In some cases, the theoretical curve, that the data are supposed to represent, should be drawn and labeled.

Many times it is necessary to put more than one curve on a graph. Curves must be drawn and labeled so that they may be easily identified and distinguished from each other. This distinction can be accomplished by

clearly labeling each curve, using a different type of line for each curve, or using different colors for each curve. Keep in mind that reproductions of graphs plotted with various colors will appear in black and white, thus losing their identity.

Much more could be said about constructing a graph, but this is more of a technique in drawing. One of the most important steps in making a good graph is to show supporting data in a legend. Such data may include, but is not limited to, the following:

1. Date.
2. Location.
3. Name of person who took the data.
4. References.
5. Simple block diagram or schematic.

This legend usually is placed in the lower-right hand areas of the graph, if possible.

RESULTS, CONCLUSIONS, RECOMMENDATIONS

It may be presumed that the most important part of keeping a logbook of the type discussed is a statement of results, conclusions, and recommendations, usually given at the end of the experiment. The entire experiment may be considered as a failure if the technician does not understand the results and cannot decide what convincing remarks should be made to express a conclusion. Conclusions should be kept brief and to the point; long, superfluous statements tend to bury the actual results of discovery. One final question might be: Has the whole significance of the results been clearly seen and expressed in the conclusions, and are the conclusions based wholly on the presentation of the results?

OTHER TYPES OF LOGBOOKS

Logbooks are used more in applications where inventions and patents are not frequently expected. Process development, experimental work, for example, is almost never patented, even though it is innovative and patentable. The reason is that it is usually easy to duplicate a process but still avoid

the patent claims. Therefore, it is usually to a company's advantage not to patent, which is also disclosing the information to the world.

There is still a need for detailed and accurate logbooks. There is frequent need to reproduce an experiment to verify information for the design engineers. If adequate records are not available, this would be difficult.

Another reason for a properly kept logbook is to have documented defense against liability suits. It is not hard to imagine a situation in a nuclear power plant, or a water treatment plant, where there are indications that something was done or not done in the plant that caused some jeopardy to human life or health. If this were <u>not</u> the case, there would be great value to the company to have dated and witnessed logbooks that proved the claims to be false. Another similar situation could arise in a pharmaceutical company. The list of examples of this type of value of a logbook would be long indeed.

Many of the same things would be done in keeping these types of logbooks as done in the one tailored for support to patents. The difference would be in emphasis.

The date and time would be just as important in these books but probably witnesses are not required. The supervisor should give guidance on this point.

There should be a statement that explains why the experiment was being done and the rationale that has led to it.

References probably are not required if they are general in nature, but they could be of value if they related very specifically to the experiment being done.

Documentation of the instruments being used is probably not required. The type of instruments would normally be obvious, or they would be indicated on the tables of data.

Layouts, diagrams, and schematics have as much value here as in the previous type. Frequently a design engineer can get much information from these, or even spot deficiencies in collection of data that need to be collected to permit a good design.

Attachment of loose articles of information, the original tabulation of data and calculations should also be in this type of logbook. The items under "Procedures" are also important here as well as the "Errors and Corrections" and "Graphs."

Results and conclusions may not always be made on a daily or even weekly basis for some types of experimental work. However, some statement about the results, or observations that indicate further experimental work should be documented frequently.

SAMPLE REPORT PAGE

Included in this module is a sample page taken from a laboratory logbook to illustrate entries required when experimental data were recorded and studies were made to substantiate a theory. Notice that the heading provides space for name, date, time, and project number. It is important to remember that, when working on different-numbered projects, a separate logbook should be kept for each project. The space marked "title" refers to the title of the experiment, not the title of the project. Normally each page of a logbook is sequentially numbered, but if it is not, you should number it as soon as your receive the logbook. The large number on the right-hand side of the page is called the registry number. This number identifies the logbook and aids in its cataloging. At the bottom of the logbook is a space for your signature and date and the signatures of any witnesses and date. After a logbook is completely filled, and no longer needed for reporting, it may be sent to a record-storage facility where it is microfilmed and kept in a safe place for retrieval at a later date.

EXAMPLE OF DATA COLLECTED IN A LABORATORY LOGBOOK

The following pages provide examples of entries made into a laboratory logbook. Shown is a modified reproduction of an actual experiment performed some years ago at a research laboratory in developing a simple photodiode laser energy monitor for a Q-switched ruby laser. Names and entries have been changed slightly for the purpose of clarity and illustration only.

Notice that the logbook entries are not necessarily performed in a rigid systematic sequence, but that the logbook contains all necessary information as previously discussed in this module.

Oscilloscope Polaroid photographs included in the actual logbook are not included in the example because of the poor quality of reproduction available.

NAME EARL J. SCRIBNER DATE 12/1/79 TIME 0945 PROJ. 4405-2

TITLE LASER PULSE DETECTOR PAGE 1 BOOK 318

ON THIS DATE CHARLES A. NELSON AND I DISCUSSED THE IDEA OF DESIGNING A SIMPLE LOW COST PHOTODIODE DETECTOR SYSTEM TO PROVIDE US WITH AN INDICATION OF THE ENERGY OUTPUT OF A LIQUID DYE Q-SWITCHED RUBY LASER.

A PHOTOGRAPH USING AN OSCILLOSCOPE CAMERA WILL BE TAKEN OF THE DETECTED PULSE AS SEEN ON THE SCREEN SUPERIMPOSED WITH A PROJECTED GRATICULE HAVING 2X2 MM SQUARES.

WE WILL USE THE INTEGRATION TECHNIQUE OF COUNTING THE NUMBER OF 2 MM SQUARES UNDER THE PULSE ENVELOPE AND THEN PLOT A CALIBRATION CURVE OF TOTAL ENERGY OUTPUT IN JOULES AS MEASURED WITH A BALLISTIC THERMOPILE VS THE NUMBER OF SQUARES COUNTED. THIS WILL BE DONE AT A SPECIFIC VERTICAL SENSITIVITY AND OSCILLOSCOPE SWEEP SPEED FOR VARIOUS CONCENTRATIONS OF DYE USED IN THE LIQUID Q-SWITCH. THIS WILL ENABLE THE OPERATOR TO PHOTOGRAPH THE PULSE, COUNT THE NUMBER OF SQUARES AND OBTAIN THE TOTAL ENERGY OUTPUT FROM THE GRAPH.

REFERENCES: EDGERTON, GERMESHAUSEN AND GRIER SD-100 APPLICATION NOTES, PP 1-8

EG&G SD-100 DATA SHEET.

WE ALSO DISCUSSED WITH LLOYD E. ALLISON THE MECHANICAL DESIGN OF THE PHOTODIODE DETECTOR HOUSING AND ITS INCORPORATION INTO THE LASER TRANSMITTER HEAD.

CALCULATIONS: WE ESTIMATE 0.5 JOULES OR APPROXIMATELY 50 MW PEAK POWER TO BE INCIDENT ON AT 45° TO A 0.006" THICK FLAT GLASS BEAM SPLITTER. SEE SKETCH.

FIG. 1

Witnessed & understood by me,	Date	Invented by	Date	Time
Sam Watanabe	12/1/79	Earl J. Scribner	12/1/79	1141

CALCULATIONS CONT'D:
WE EXPECT REFLECTED POWER TO BE: $50\,MW \times 0.08 = 4\,MW$

ACCORDING TO EG&G THE MAX. POWER DENSITY INCIDENT ON SD-100 SURFACE IS 5, 9 W/CM² OR 432 MW FOR 0,01 IN² ACTIVE AREA.

BEAM AREA $= \pi r^2 = \pi\left(\frac{1}{4}\right)^2 \simeq \frac{3}{16}\ IN^2$

$$\text{WATTS/AREA} = \frac{4\,MW}{(3/16)\,IN^2} = \frac{21.3\,MW}{IN^2}$$ REFLECTED BEAM POWER DENSITY

BEAM POWER DENSITY x AREA SD-100 $= \frac{21.3\,MW}{IN^2} \times 0.01\,IN^2$

$= 0.213\,MW$

THIS IS $\simeq 0.5 \times 10^6$ GREATER THAN MAX. ALLOWED SO AN N.D. FILTER OF 6.0 WILL GIVE US:

$10^{-6} \times 2.13 \times 10^5\,W$ OR 0,213 WATTS INTO DIODE

A NARROW-BAND INTERFERENCE FILTER HAVING A CENTER WAVELENGTH OF 6943Å, $\simeq$ 50% TRANSMISSION, AND 30Å BANDWIDTH AT THE 50% PTS WILL BE PLACED IN THE OPTICAL PATH TO IMPROVE SIGNAL TO NOISE RATIO AS SOME PUMP LIGHT WILL BE ALSO REFLECTED OFF BEAM SPLITTER INTO DIODE. WE HAVE A SPECTROLAB FILTER ON HAND WITH THE ABOVE SPECIFICATIONS. N.D. FILTERS AND DIFFUSERS WILL BE ADDED TO REDUCE BEAM INTENSITY AND REMOVE HOT-SPOTS IN BEAM.

EXPECTED SIGNAL LEVEL OUTPUT INTO 50-OHM LOAD IS:

FOR $S_{\lambda=6943A} = 0.25\,A/W$ (EG&G DATA SHEET)

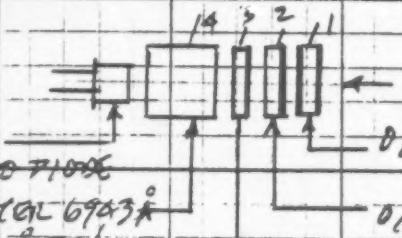
$V_o \simeq 0.25\frac{A}{W} \times 0.2\,W \times 50\,\Omega = 2.5\,VOLTS$

Witnessed & understood by me,	Date	Invented by	Date	Time
Sam Watanabe	12/1/79	Earl J. Scribner	12/1/79	1642

NAME EARL J. SCRIBNER DATE 12/2/79 TIME 0945 PROJ. 4405-2

TITLE LASER PULSE DETECTOR PAGE 3 BOOK 318

BEFORE CONSTRUCTION ON THE ACTUAL MOUNT WAS STARTED, A PHOTODIODE WITH THE FOLLOWING LAYOUT WAS SET UP IN THE LABORATORY. DIODE WILL BE REVERSED BIASED (PHOTO CONDUCTIVE MODE) AT 100 VOLTS DC AND SIGNAL OUTPUT TERMINATED INTO 50-OHMS AT VERTICAL AMPLIFIER INPUT (TYPE L) OF 555 TEKTRONIX OSCILLOSCOPE,



REQUIRE ≅ 6,0 ND ATTENUATION,

0,067" THK GLASS DIFFUSER FROSTED EACH SIDE,

0,055" THK ND 4,0 FILTER FROSTED FNT. SIDE

0,055" THK ND 1,0 FILTER FROSTED FNT. SIDE

	PATH LOSS
1ST	0,3
2ND	9,0
3RD	1,3
4TH	0,3
TOTAL ND =	5,9

FIG. 2 OPTICAL PATH ENTRANCE TO DIODE HD,

A HOLLOW TUBE WAS ROUGHED OUT TO HOLD THE DETECTOR, FILTER AND ATTENUATORS. A CONNECTOR WAS ATTACHED ON ONE END TO APPLY THE BIAS AND ANOTHER FOR THE SIGNAL OUTPUT LEAD. THE LASER WAS FIRED INTO THE GLASS BEAM SPLITTER AT 45°.

TEST CONDITIONS: LASER WILL BE OPERATED AT 2.6 KV USING .495 μF STORAGE CAPACITOR IN SERIES WITH A 175 μh AIR CHOKE TO PUMP 2 PEK XE 2040 XENON LAMPS CONNECTED IN SERIES. SHOTS WILL BE MADE AT 4-MIN. INTERVALS AFTER WARM UP. 3 DYE CONCENTRATIONS WILL BE USED 1:40, 1:35, 1:25 VPC IN $CHCl_3$. ENERGY OUTPUT IN MAIN BEAM WILL BE MEASURED AND RECORDED WITH THERMOPILE AND MICROVOLTMETER. PHOTOGRAPH OF DETECTED PULSE WILL BE MADE AND TOTAL NUMBER OF SQUARES ~~WILL BE COUNTED~~ UNDER PULSE ENVELOPE WILL BE COUNTED. SEE BLOCK DIAGRAM FOLLOWING PAGE.

Witnessed & understood by me,	Date	Invented by	Date	Time
Sam Watahara	12/2/79	Earl J. Scribner	12/2/79	1655

NAME EARL J. SCRIBNER DATE 12/3/79 TIME 0815 PROJ. 4405-2

TITLE LASER PULSE DETECTOR PAGE 4 BOOK 318

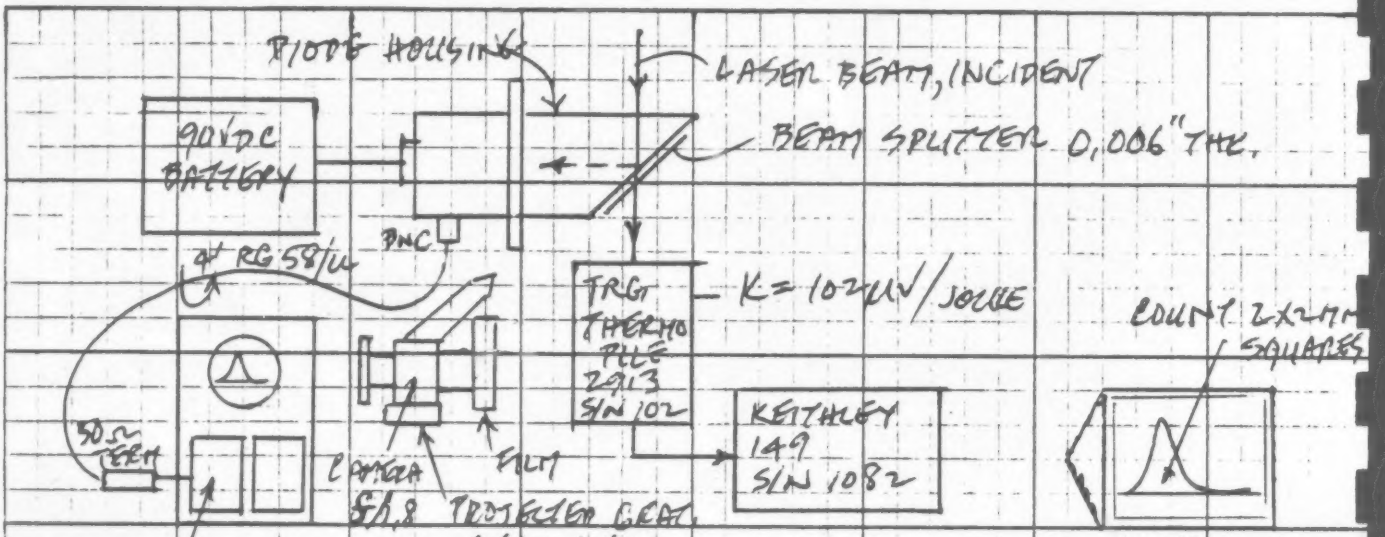


FIG. 3 BLOCK DIAGRAM OF MEASUREMENT SYSTEM

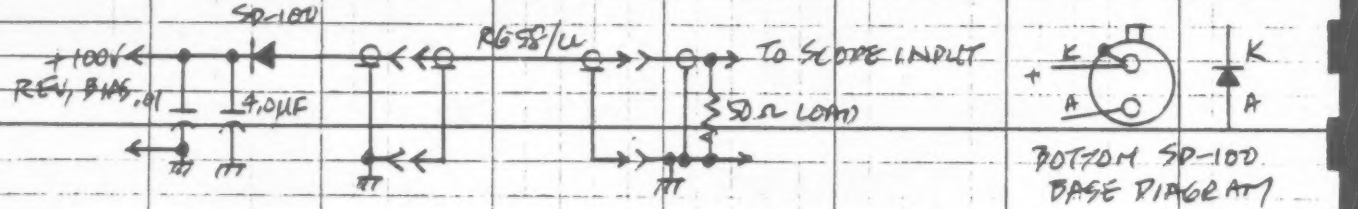


FIG. 4 DIODE POWER & SIGNAL CONNECTIONS

AFTER MANY TRIAL FIRINGS, THE NEUTRAL DENSITY FILTERS WERE ADJUSTED TO OBTAIN A 1-2 VOLT PEAK AMPLITUDE SIGNAL SINCE TOTAL ENERGY OUTPUT OF LASER WAS LESS THAN EXPECTED. INPUT OPTICS NOW CONTAIN A 3.40 ND FILTER, NB FILTER, 1.067" IN THK DIFFUSER DISC. SCOPE SETTINGS ARE 0.5V/CM, 100NS SWEEP SPEED. DATA IS TABULATED AND PLOTTED ON FOLLOWING 6 PAGES.

187

Witnessed & understood by me,	Date	Invented by	Date	Time
Sam Watanabe	12/3/79	Earl J. Scribner	12/3/79	1405

NAME EARL J. SCRIBNER DATE 12/3/70 TIME 1415 PROJ. 4405-2

TITLE LASER PULSE DETECTOR PAGE 5 BOOK 318

TABLE-1 MEASURED ENERGY VS NO. SQUARES

SHOT NO	TIME	UV OUT K=102 uv/f	JOULES	NO. SQ.	DYE CONC.
1	1034 AM	8.9	.087	13	1:40 VPC
2	1038	8.6	.084	11	IN CHCl3
3	1042	9.8	.096	14	
4	1046	9.3	.091	13	
5	1050	9.1	.089	12	
6	1054	8.8	.086	13	
7	1158	14.4	.141	16	1:35 VPC
8	1202 PM	12.2	.119	15	IN CHCl3
9	1206	14.4	.141	17	
10	1210	14.1	.138	17	
11	1214	15.3	.150	18	
12	1218	15.0	.147	18	
13	1222	18.7	.183	23	1:25 VPC
14	1226	22.9	.224	26	IN CHCl3
15	1230	21.4	.209	25	
16	1234	21.9	.214	24	
17	1238	20.8	.204	23	
18	1242	19.6	.192	22	

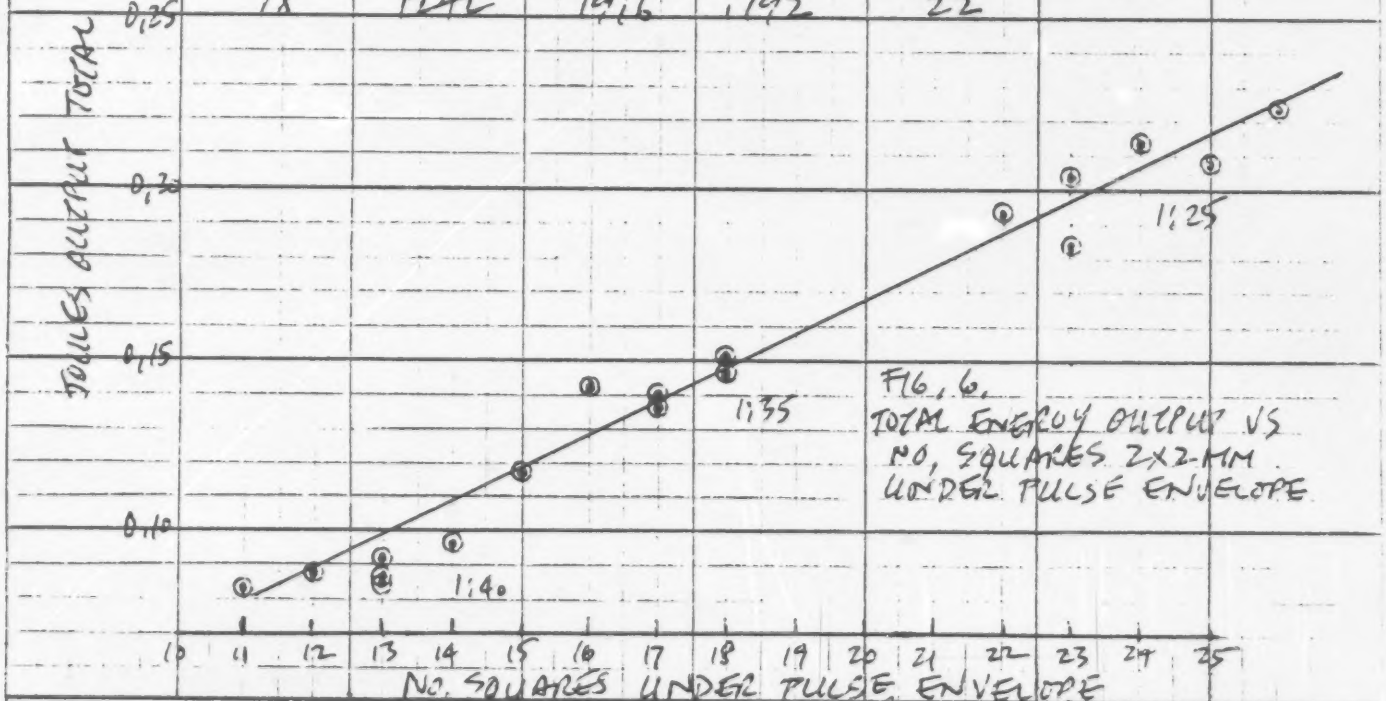


FIG. 6. TOTAL ENERGY OUTPUT VS NO. SQUARES 2X2-MM UNDER PULSE ENVELOPE

Witnessed & understood by me,	Date	Invented by	Date	Time
Sam Watanabe	12/3/70	Earl J. Scribner	12/3/70	1735

NAME EARL J. SCRIBNER DATE 12/4/79 TIME 1315 PROJ. 4405-2

TITLE LASER PULSE DETECTOR PAGE 6 BOOK 318

CONCLUSIONS AND RECOMMENDATIONS:

IT APPEARS THAT THE TECHNIQUE OF MEASURING THE TOTAL ENERGY OUTPUT OF A Q-SWITCHED RUBY LASER BY DETERMINATION OF THE AREA FOUND UNDER THE MEASURED AND PHOTOGRAPHED PULSE IS FEASIBLE BUT LEAVES MUCH TO BE DESIRED AS INDICATED BY THE GRAPH. VARIATIONS AS MUCH AS 30% ARE SEEN AT VARIOUS DYE CONCENTRATIONS. IT IS EVIDENTLY RATHER DIFFICULT TO VISUALLY GET AN ACCURATE COUNT OF THE EXACT NUMBER OF 2 mm SQUARES UNDER THE PULSE ENVELOPE. THE ACTUAL PULSE BEING ~~TESTED~~ LESS THAN 2 CM IN HT. AND 1/3 CM IN WIDTH FOR BETTER ACCURACY ONE WOULD HAVE TO INCREASE THE SIZE OF THE PULSE ON THE SCREEN - THIS IS DIFFICULT TO ACHIEVE AS WE BEGIN TO LOSE THE IMAGE BECAUSE OF THE HIGH WRITING RATES INVOLVED.

IT IS SUGGESTED THAT AN ELECTRONIC TYPE RC PASSIVE OR ACTIVE INTEGRATOR BE EMPLOYED WITH A SAMPLE AND HOLD CIRCUIT AND A ZERO RESET DEVICE. THIS WOULD IMPROVE THE ACCURACY AND REPEATABILITY OF MEASUREMENTS BUT MAY ADD SUBSTANTIALLY TO THE COST OF IMPLEMENTATION DISCUSSION WILL CONTINUE WITH PROJECT LEADER.

Witnessed & understood by me,	Date	Invented by	Date	Time
Sam Watanabe	12/4/79	Earl J. Scribner	12/4/79	1702

Page 16/S-9

NAME _____ DATE _____ TIME _____ PROJ. _____

TITLE _____ PAGE _____ BOOK 318

Witnessed & understood by me,	Date	Invented by	Date	Time

LABORATORY REPORT EVALUATION[1]

The laboratory report bears direct witness to the quality of work accomplished in the experiment. Therefore, the student should strive to develop quality lab reports. As an aid in evaluating your report, the following grading scale--used by many instructors--is included. A laboratory report grade of 9-10 is excellent; a laboratory report grade of 4-5 indicates an inadequate report.

9-10 AN EXCEPTIONAL REPORT. All questions have precise and accurate answers. The calculations are neat and easily checked. The student has done more than the laboratory manual requires taking extra data or analyzing the data in a new and different way. The report contains no errors and the results of the experiment are as accurate as the equipment allows.

8-9 AN EXCELLENT REPORT. All questions have precise and accurate answers. The calculations are neat and easily checked. The student has done all that the laboratory manual requires. The report contains no errors and the results are as accurate as the equipment allows.

7-8 A GOOD REPORT. All questions have adequate answers. The calculations are readable and can be checked. The student has done all that the laboratory manual requires. The report contains few, if any, errors but the results are not as accurate as the equipment allows.

6-7 AN ACCEPTABLE REPORT. All of the questions have correct answers. The calculations are shown and may be checked with difficulty. The student has done all that the laboratory manual requires. The report contains some errors. The results are not nearly as accurate as the equipment allows.

5-6 A POOR REPORT. Some of the questions have correct answers. Most of the calculations are shown and they may be checked with difficulty. The student has done most of the required laboratory work. The report contains errors and the results are inaccurate.

4-5 AN INADEQUATE REPORT. The questions have incorrect answers. Some of the calculations are shown but cannot be checked. The student has done some of the required work. The report contains many errors. The results contain serious errors.

The overall grade on a report may deviate from these standards depending on the individual parts of the report and the student's performance in the laboratory. Students with poor laboratory skills or those who are not prepared to do the lab generally receive lower scores.

---

[1] Credit for this grading scale belongs to Dr. W. Duetschman, Oregon Institute of Technology.

LABORATORY

Select one or both of the following experimental activities depending on the availability of suitable facilities, condition, and time. Prepare logbook records for the entire experiment, make graphical plots as needed, and draw conclusions. The student should get a lab logbook and use it for other exercises in this course.

MATERIALS

Laboratory logbook sheets
Thermometer
Air conditioned/heated room with thermostat
Clock or stopwatch

PROCEDURES

Energy saving by raising the thermostat setting in warm weather and lowering the setting in cold weather. (This experiment is probably impractical if the difference in indoor and outdoor temperatures are less than 15°F.)

1. Set the thermostat at 70° (summer) or 80° (winter).
2. Record the room temperature every two minutes until the cooling (heat) goes off. Continue measuring the room temperature every two minutes until the cooling (heat) comes on again. Continue recording the time and temperature through two or three cycles (on and off).
3. Then set the thermostat to 78° (summer) or 72° (winter) and repeat the above procedures through two or three cycles at the second temperature.
   a. What can be said about the cycle time at the two temperatures?
   b. Since air conditioning or heating only uses energy when the unit is running, what can be said about room temperature and energy savings (costs)?
   c. How much did the temperature overshoot the set-point (thermostat setting)?
   d. What can be said about the rate of temperature rise (summer) or temperature drop (winter)?

MATERIALS

Electric fan

2 Thermometers

Foamed polystyrene cup

Glass or metal cup - These two cups should be about the same size.

PROCEDURES

HEAT LOSS FROM AN OBJECT

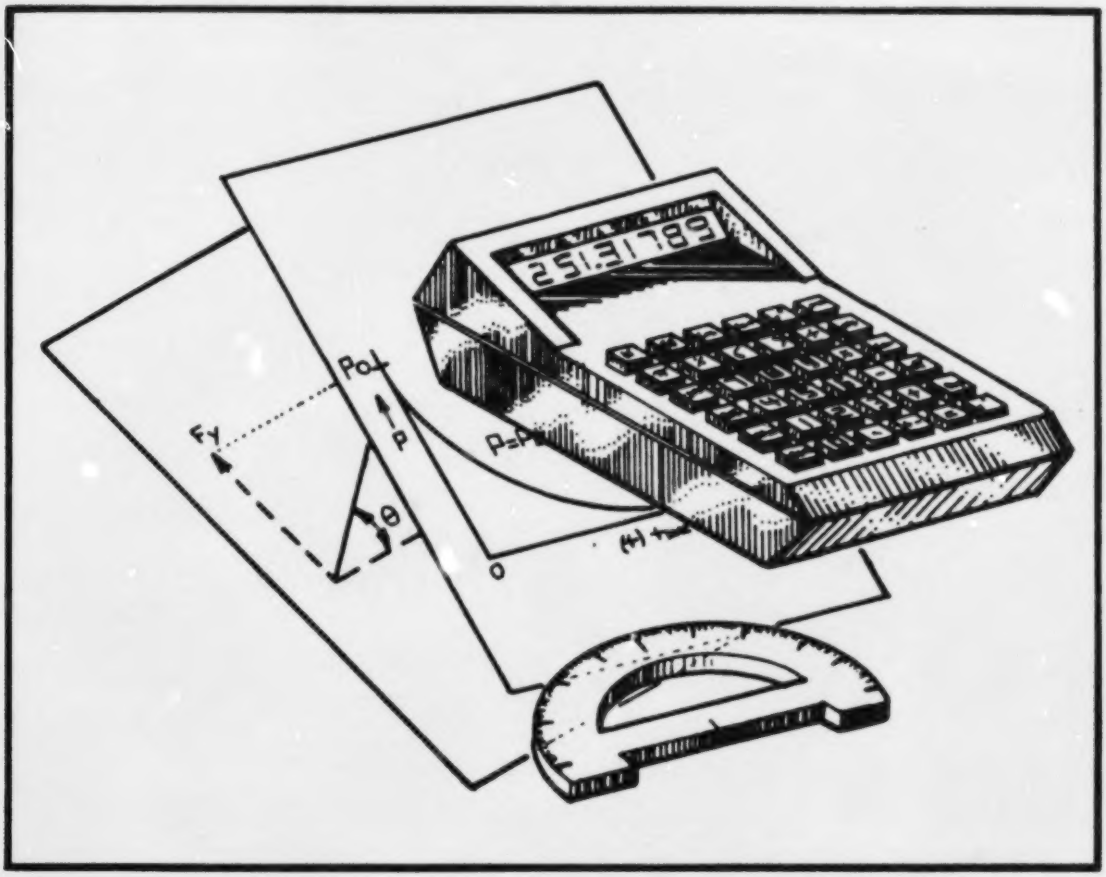
1. Fill both cups with hot water and place a thermometer in each out of all drafts. After a minute read both thermometers (they should read approximately the same temperature). Read the thermometers every five minutes for 30 minutes.
2. Repeat the experiment, but place both containers about the same distance from and near the front of the operating fan. Make the same time-temperature measurements.
   a. What can be said about the relative heat loss from the two containers?
   b. What can you conclude about the effect of the fan breeze on the heat loss?
   c. Does the foamed polystyrene provide much value in reducing heat loss if the containers are in a breeze?

UTC

UNIFIED TECHNICAL CONCEPTS

MODULE S-10

SINUSOIDAL WAVES



SUPPLEMENTARY

CORD CENTER FOR OCCUPATIONAL RESEARCH AND DEVELOPMENT

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INTRODUCTION

Certain physical quantities change with time or distance in a periodic manner and their behavior can be described mathematically by the sine function. Periodic motions of this type include oscillations of springs and pendula, propagation of sound and light energy and traveling disturbances in strings. In each case, the mathematical equation which describes the wave relates the wave displacement to the variation of time and/or distance, and employs certain important wave parameters such as frequency, wavelength, wave velocity, and phase. The phase relationships between two or more sinusoidal waves of the same frequency are important in dealing with interference and coherence of wave motion, and in the treatment of ac electrical circuits.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Recognize wave equations which are time-dependent, space-dependent, or both space-and-time dependent, and relate the important wave parameters to the constants in the equations.
2. Given two or more wave parameters, determine all the others from the equations which relate them.
3. Determine the phase relationship between two sinusoidal waves using terms like "leading" and "lagging," and display the waves using rotor diagrams.

TIME-DEPENDENT HARMONIC OSCILLATION

Consider a physical quantity that varies periodically with time in a regular manner, such as the position of a mass oscillating on a spring, or the voltage of an alternating current electrical circuit. Such time variations can be expressed by the sine function and are called **harmonic** oscillations.

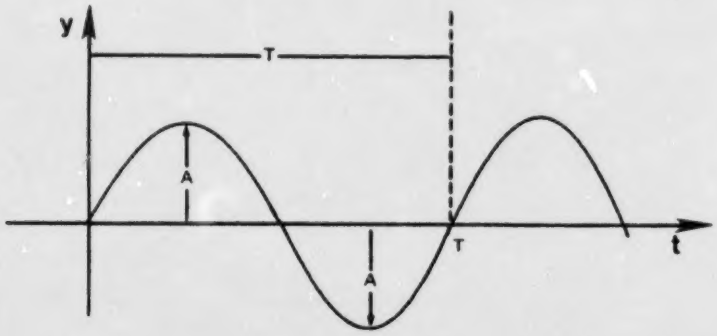


Fig. 1 Harmonic oscillation: A sine wave in time.

Figure 1 shows a graphical representation of such a harmonic oscillation, a simple sine wave. The independent variable is time; the physical quantity varying with time is represented by y. Mathematically, this curve is expressed by the equation:

$$y = A \sin \left[\frac{2\pi}{T} t \right] \qquad \text{Equation 1}$$

The equation written in this way is especially convenient because it displays two important properties which distinguish one sine wave from another. These are A, the amplitude of the wave, and T, the period of the wave. The amplitude of the wave is the maximum value of y in either the positive or negative direction, as shown in the figure. Thus, it represents the maximum displacement of the wave. This fact is also verifiable from Equation 1. The maximum value of y will be the amplitude A times the maximum value of the sine function as t varies. Recalling that the maximum value of the sine function is 1 in both positive and negative directions, it follows that:

$$y_{max} = A\ (\pm 1)$$

$$y_{max} = \pm A \qquad \text{Equation 2}$$

Now notice that when t = 0, y = 0 because y = A sin (0) = 0. Thus the displacement y of the wave is zero at the origin of Figure 1. When t = T, y = A sin 2π = 0, and the angle--the argument of the sine function--has passed through one complete revolution of 2π radians. This particular value of t=T is also shown on the figure. As t increases further, the sine function simply repeats itself over and over. Notice that the wave passes through zero also at the midpoint where y = A sin π = 0. Thus T, or the period of the wave, represents the time required to execute one complete cycle.

EXAMPLE A: PERIOD AND AMPLITUDE OF A SINUSOIDAL WAVE.

Given: The ac voltage in a circuit varies sinusoidally in such a way that the maximum voltage is 170 volts and each cycle repeats every 1/60 second.

Find: The equation which represents the variation of voltage with time.

Solution: Evidently in this case: $A = V_{max} = 170$ volts

$T = 1/60$ sec

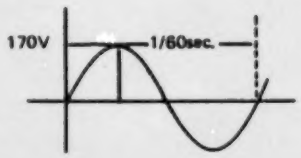
Substituting into Equation 1, where y is written as a voltage:

$$V = 170 \sin \frac{2\pi}{(1/60)} t$$

or

$$V = 170 \sin(120\pi t)$$

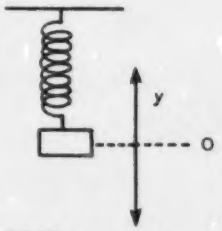
where V is in volts and t in seconds.



EXAMPLE B: PERIOD AND AMPLITUDE OF A SINUSOIDAL WAVE.

Given: The displacement y of a mass oscillating on a spring, measured from the equilibrium (at rest) position is given by:

$$y = 20 \sin 4\pi t$$



where y is in centimeters and t in seconds.

Find: The amplitude and period of the oscillating mass.

Solution: Comparing the equation given with the standard form, Equation 1, it is possible to match:

$A = 20$

$$\frac{2\pi}{T} = 4\pi \quad \text{or} \quad T = \frac{2\pi}{4\pi} = \frac{1}{2}$$

Thus $A = 20$ cm and $T = 1/2$ second

The period T of a wave is often described by its inverse $1/T = f$, where f is called the frequency of the oscillation. Clearly, if $T = 1/2$ second, as in Example B, then $f = 2$. In other words if the time for one oscillation is 1/2 second, then the number of oscillations completed in one second is the frequency 2. Thus the frequency of an oscillation is simply the number of complete oscillations executed in one second. The unit for frequency is often written as "Hertz," which is the same as "oscillations per second." The frequency of household ac voltage is 60 Hz or 60 complete oscillations every second, as in example A.

Since $T = 1/f$, Equation 1 can also be written:

$$y = A \sin 2\pi ft \qquad \text{Equation 3}$$

Finally, the quantity $2\pi f$ is called the "angular frequency" and symbolized by the Greek letter omega (ω). The angular frequency ω is measured in radians per second. The argument of the sine function is then simply ωt in radians so Equation 3 becomes:

$$y = A \sin \omega t \qquad \text{Equation 4}$$

Example C illustrates the use of Equation 4.

EXAMPLE C: FREQUENCY OF A SINUSOIDAL OSCILLATION.

Given: An ac current varies sinusoidally according to the equation:

$$i = i_{max} \sin \omega t$$

where i_{max} = 20 amperes and ω = 251.3 radians/second

Find:
1. The frequency and period of the oscillation.
2. The value of the current when t = 0, t = 0.01 sec, t = 0.02 sec, and t = 0.025 sec.

Solution:

1. Since $\omega = 2\pi f$, $f = \frac{\omega}{2\pi} = \frac{251.3}{2\pi} = 40$ Hz

$$T = \frac{1}{f} = \frac{1}{40} = 0.025 \text{ sec}$$

2. The magnitude of the current can be found at any time t by substitution into the equation given, with values of i_{max} and ω inserted. Remember that the argument of the sine function is in radians.

$$i = 20 \sin (251.3\ t)$$

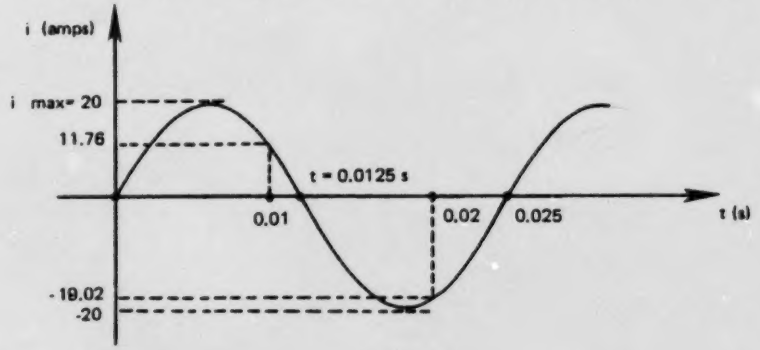
at t = 0, $i = 20 \sin (0) = 0$ amps

at t = 0.01, $i = 20 \sin (2.513) = 11.76$ amps

at t = 0.02, $i = 20 \sin (5.026) = -19.02$ amps

at t = 0.025, $i = \sin (6.282) = 0$ amps

Note: In the last case, rounding off errors may not yield exactly zero, but it is clear that the sine wave must again be zero when t = T = 0.025 sec.



SPACE-DEPENDENT SINUSOIDAL OSCILLATION

The preceding section describes a sine wave in time, or a time-dependent sinusoidal oscillation. It is often necessary to deal with a sine wave in space, where the independent variable is distance along, say, the x-axis. The simplest example is the wave set up in a rope when one end is shaken in a regular manner. If a snapshot were taken of the rope, the wave would be "frozen" in time, and the displacement of the rope from the center line would vary as a function of the distance x along the rope. The situation now is in exact analogy with the case described previously. The wave is expressed by the equation:

$$y = A \sin \left[\frac{2\pi}{\lambda} x\right] \qquad \text{Equation 5}$$

and is drawn in Figure 2.

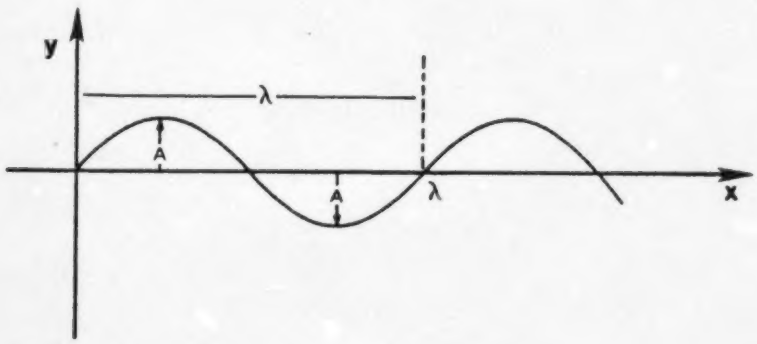


Fig. 2 Sine wave in space.

The variable x now plays the role of the variable t, and the wavelength λ plays the same role as the period T. Evidently the wavelength λ represents the distance along the wave equal to one complete oscillation, as shown in Figure 2. In this case too, the quantity $2\pi/\lambda$ is often written as a single quantity κ, called the wave number, or the propagation constant. Just as previously $\omega = 2\pi/T$, we define:

$$\kappa = \frac{2\pi}{\lambda} \qquad \text{Equation 6}$$

so that the wave can be written in the form:

$$y = A \sin \kappa x \qquad \text{Equation 7}$$

EXAMPLE D: SINUSOIDAL WAVE IN SPACE.

Given: The wave form which appears on the surface of a lake at a certain instant can be described by the equation:

$$y = 2 \sin \left(\frac{\pi}{10} x\right)$$

where x and y are both in feet.

Find:
1. The wavelength and amplitude of the wave.
2. The distance between consecutive crests or consecutive troughs of the wave.

Solution:
1. By comparison with Equation 5,

$$\frac{2\pi}{\lambda} = \frac{\pi}{10}, \text{ so } \lambda = \frac{10}{\pi} \cdot 2\pi = 20$$

Therefore, $\lambda = 20$ ft and

$A = 2$ ft

2. These distances correspond to the wavelength of the wave, or 20 feet.

TRAVELING WAVE

The wave form assumed by a rope when one end is shaken up and down periodically seems to be traveling from one end toward the other, even though each particle in the rope simply moves up and down. Evidently the displacement of any small piece of the rope depends both on its position along the center line (x) and on the time (t). To represent such a traveling wave, the Equations 4 and 7, presented in the preceding sections are not adequate because each one depends on one variable alone. However, it is possible to combine them in such a way that the displacement y is a function of both time and position, as follows:

$$y = A \sin (\kappa x \pm \omega t) \qquad \text{Equation 8}$$

In this equation, the (+) sign represents a wave moving to the left (negative direction along x-axis) and the (-) sign represents a wave moving to the right (positive direction along the x-axis). The wave cannot be plotted on a 2-dimensional plot because it now contains 3 variables: x, t and y. However notice that if either x or t is chosen as some fixed value, there are only two variables, and the equation can be plotted as before. In particular, if $x = 0$, the time variation of the wave is described at the fixed point of the origin and is then like Equation 4. On the other hand, if $t = 0$, the space variation of the wave, "frozen" at the instant $t = 0$, is described by the equation, and is then like the Equation 7. Thus Equation 8 is a powerful equation, including changes in the shape of the wave in both time and space.

The velocity v of any such traveling wave is found from

$$v = \lambda f \qquad \text{Equation 9}$$

Equation 9 is reasonable since, for example, if a wave has a frequency of 2 oscillations per second, and each oscillation represents a displacement in space, or wavelength, of say 3 feet, it will travel at a rate of 3 feet/oscillation x 2 oscillation/second or 6 feet/second.

EXAMPLE E: TRAVELING WAVE.	
Given:	One end of a long stretched rope is mechanically moved up and down regularly 10 times per second. A sinusoidal traveling wave of amplitude 6 inches moves along the rope with a speed of 40 feet/sec in the positive x-direction.
Find:	1. Frequency, angular frequency, period, wavelength, wave number of the traveling wave. 2. Equation of the traveling wave specified for this case. 3. Displacement of the rope at the origin and when $t = 0$; at a position 3 ft from the origin 1 second after the oscillations are begun.

Solution: 1. The frequency of the oscillations is determined by the driving mechanism at one end, that is, f = 10 osc/sec = 10 Hz.

$$\omega = 2\pi f = 62.83 \text{ radians/sec}$$

$$T = 1/f = 1/10 = 0.1 \text{ sec}$$

$$\lambda = v/f = \frac{40 \text{ ft/sec}}{10 \text{ osc/sec}} = 4 \text{ ft}$$

$$\kappa = \frac{2\pi}{\lambda} = \frac{2\pi}{4 \text{ ft}} = 1.571/\text{ft}$$

2. Since A = 0.5 ft, substituting into Equation 8,

$$y = A \sin (\kappa x \pm \omega t)$$

$$y = 0.5 \sin (1.571 x - 62.83 t)$$

where x and y are in ft, t in seconds.
Notice that the (-) sign is selected to agree with the given specifications of wave motion in the (+) x-direction.

3. At t = 0 and x = 0, in the preceding Equation,

$$y = 0.5 \sin (0) = 0$$

Thus the wave has zero displacement at the origin when the motion there begins.

At t = 1 sec and x = 3 ft,

$$y = 0.5 \sin [1.571 (3) - 62.83 (1)]$$

$$y = - 0.5 \text{ ft}$$

The rope at 3 ft from the origin is displaced 0.5 ft below the center line (x-axis) at the instant 1 second after the motion begins.

PHASE

Up to this point, all the wave equations considered have been such that the displacement was zero when the initial conditions were zero, that is $x = 0$ and $t = 0$. Thus the sine wave was always zero at the origin of coordinates. This need not always be the case. A timer might be turned on ($t = 0$) at the instant when a wave already has some displacement as in Figure 3. The dashed line shows the continuation of the sine curve before the clock was turned on. Clearly $y \neq 0$ at $t = 0$. The wave equation must be modified

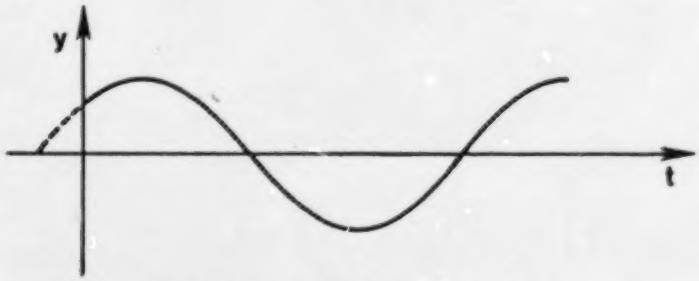


Fig. 3 Wave with initial displacement.

slightly to include such cases, as in the time-dependent oscillation:

$$y = A \sin (\omega t + \phi) \qquad \text{Equation 10}$$

Here ϕ is called the initial phase angle. The angle ϕ, as shown, is a term added to the argument of the sine function. If ϕ is zero, Equation 10 is identical to Equation 4, and the wave has zero displacement at time zero. When $\phi \neq 0$, however, the argument of the sine function is clearly not zero, even at $t = 0$. Thus the initial phase angle ϕ permits an initial displacement y at $t = 0$.

Consider the following examples for a wave with $\omega = 2\pi$ rad/second.

For $\phi = 0$	$y = A \sin (2\pi t)$	Equation 11
For $\phi = \pi/6$	$y = A \sin (2\pi t + \pi/6)$	Equation 12
For $\phi = \pi/2$	$y = A \sin (2\pi t + \pi/2)$	Equation 13
For $\phi = \pi$	$y = A \sin (2\pi t + \pi)$	Equation 14

The sine waves, which differ only in phase angle, are plotted in Figure 4.

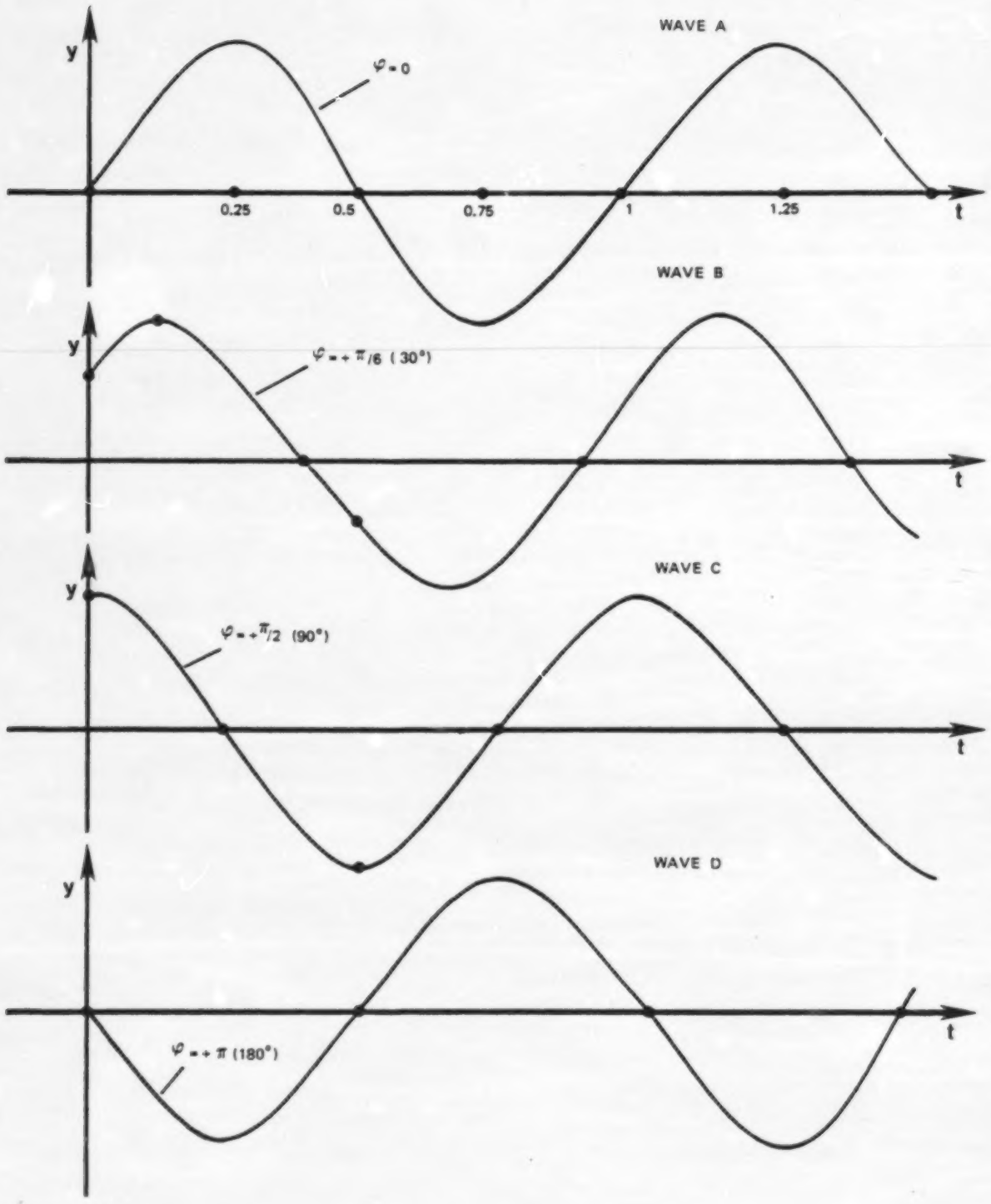


Fig. 4 Sine waves with various initial phases.

Examination of the four sine waves shows that the phase angle ϕ produces a shift of the wave such that the wave appears to get a greater head start the greater the angle ϕ. For example, in graph B, the maximum positive peak is reached sooner (for a smaller t), than in graph A. In successive graphs C and D this shift continues. It is interesting to note that graph C actually is a plot of the cosine function. Thus a sine wave with an initial phase angle of 90° or $\pi/2$ radians is identical to a cosine wave. The only difference between these two waves is the displacment at t = 0. For the sine wave, displacement at origin is zero; for the cosine wave it is a maximum, or equal to the amplitude A. This situation is simply a result of the trigonometric identity:

$$\sin (\theta + 90°) = \cos \theta$$

Notice also that when the phase angle is π--wave D--the sine wave has shifted by one-half period so that valleys appear where crests originally were, and vice versa. Whereas any two of the waves can be said to be "out-of-phase" because crests and valleys do not coincide, waves A and D represent the extreme cases and are said to be "completely out-of-phase."

It is also common terminology to say that wave B "leads" wave A by $\pi/6$ radians or 30°, or that wave C "leads" wave A by $\pi/2$ radians of 90°. This is exactly the same thing as saying that wave A "lags" behind wave B by 30°, or that it "lags" behind wave C by 90°. Thus the fact that sinusoidal waves differ in initial phase angle produces the result that one is shifted relative to the other and is said either to "lead" or to "lag" behind the other.

If in the equations 11 - 14, the phase angles were negative angles, then the waves B, C, and D would be shifted in the other direction and would lag behind wave A by progressively larger amounts. Summarizing, the wave equation:

$$y = A \sin (\omega t \pm \phi)$$

is a wave which <u>leads</u> or <u>lags</u> behind the wave with $\phi = 0$, depending on whether ϕ is respectively a <u>positive</u> or a <u>negative</u> angle.

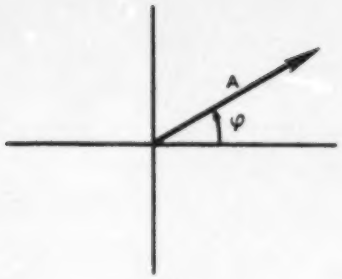


Fig. 5 Rotor diagram of a sinusoidal wave.

An interesting and helpful way of picturing sinusoidal waves differing in initial phase and amplitude is with the rotor diagram. Each wave is represented by a vector, with vector magnitude equal to the wave amplitude and with vector "direction" determined by the initial phase ϕ. Figure 5 illustrates a general case: $y = A \sin(\omega t + \phi)$. Using this technique, the four waves of Figure 4 would be drawn as shown in Figure 6.

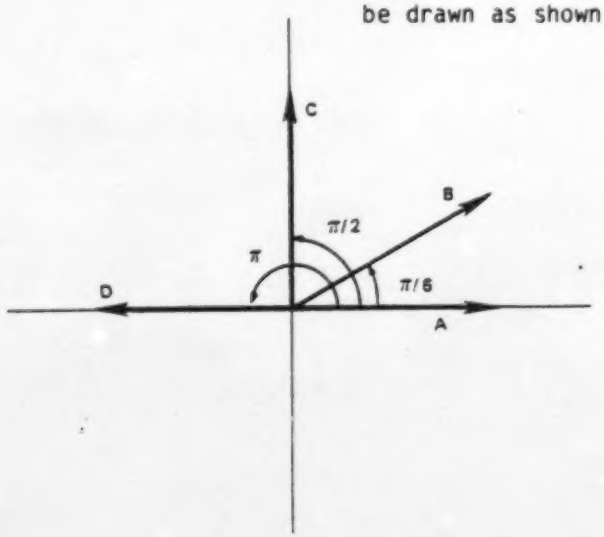


Fig. 6 Rotor diagram of equal amplitude waves.

Such rotor diagrams are especially helpful when analyzing ac circuits. It turns out that the voltage and current in an ac circuit are both sinusoidal (harmonic) oscillations, but may appear with a relative phase angle between them. For example, if the current is expressed as $i = i_{max} \sin \omega t$, the voltage is in general $v = v_{max} \sin(\omega t + \phi)$. Figure 7a illustrates a common situation in which the voltage <u>lags</u> behind the current by 90°. Figure

7b illustrates another situation in which the voltage leads the current by 90°. The "rotors" need not have the same amplitude or length.

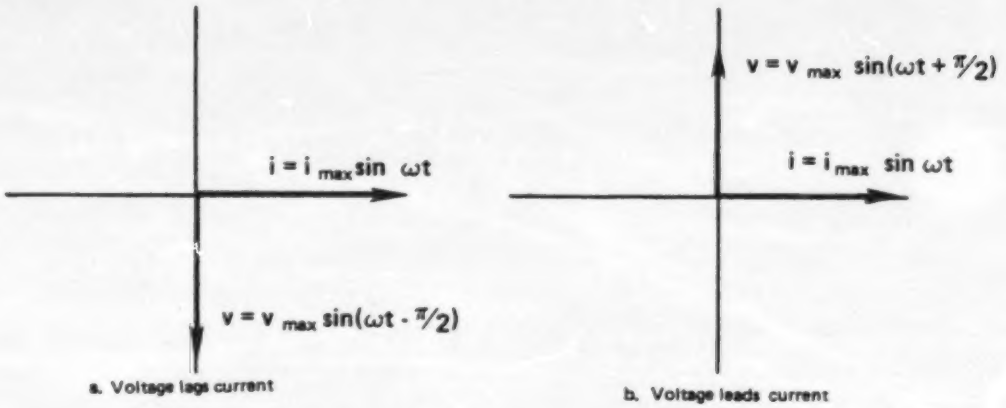


Fig. 7 Rotor diagrams for ac voltage and current.

Finally, consider Figure 8, which shows a number of waves in-phase and another number of waves out-of-phase. These waves could represent any kind of wave motion, including sound or light. The equations developed in this module apply to any kind of waves, the only difference being the physical meaning given to the displacement y.

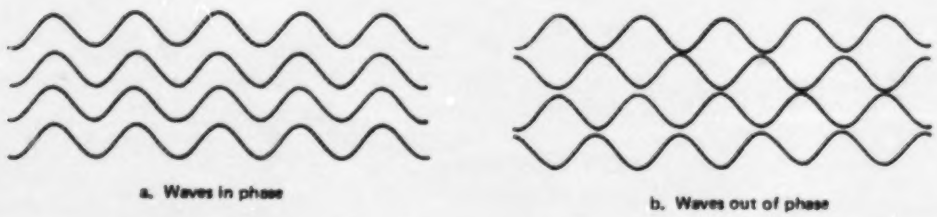


Fig. 8 Phase relationships between a number of waves.

How do the wave equations representing the waves in-phase differ from those representing waves out-of-phase? Evidently when waves are in-phase, their valleys and peaks correspond at all places and times--they are "in

step." For this to happen the entire argument of the sine function of a traveling wave must be the same for each of the member waves at every value of x and t. That is, in the traveling wave,

$$y = A \sin (\kappa x - \omega t + \phi) \qquad \text{Equation 15}$$

which now includes an initial phase angle ϕ, the entire argument $(\kappa x - \omega t + \phi)$ must be the same for all waves of the set in order that each has the identical value for y at a given x and t. Because of the importance of the entire argument, it is called the "total phase" of the wave. Notice that the inital phase ϕ is therefore only a part of the total phase. When several waves have the same "total phase," they are said to be "in-phase," as in Figure 8; otherwise they are "out-of-phase" to one degree or the other. Laser light in particular has the "in-phase" property of the waves shown in Figure 8, a property which is called, in general, "coherence." Coherent waves do not lead or lag one another. They are always "in step."

TEST

1. Given the wave equation

   $$y = 5 \sin (0.4\pi x - \pi), \quad x \text{ and } y \text{ in ft},$$

   determine with appropriate units the following:

   a. Amplitude.
   b. Wavelength.
   c. Wave number.
   d. Initial phase angle.
   e. Displacement at x = 0; at x = 1 ft.

2. Given the traveling wave directed along the z-axis

   $$y = 100 \sin (20\pi z + 1000\, t + \pi/2),$$

   with y and z in cm and t in seconds. Determine the following with appropriate units:

   a. Wavelength.

b. Frequency.

c. Wave velocity and direction.

d. Amplitude.

e. Angular velccity.

f. Wave number.

g. Period.

h. Displacement at $z = 0$, $t = 0$; at $z = 0$, $t = 2$ sec; at $z = 5$ cm, $t = 0$; at $z = 10$ cm, $t = 3$ sec

3. Given a wave equation given by

$$y = 25 \sin 120\pi t:$$

a. Write an equation which represents a similar wave except that it leads by 60°; lags by $\pi/4$ radians; leads by 360°; lags by π radians.

b. Draw a rotor diagram exhibiting the original and all four of the waves described in (a). Assume all have the same amplitude.

c. Sketch the waves as a function of time, showing the relative shifts involved.

4. Write the wave form equation for a sinusoidal oscillation for each of the following:

a. In <u>time</u>, given a period of 5 sec, a maximum displacement of 15 cm, and an initial phase angle of + 30°.

b. In <u>space</u>, given a wave number of $\pi/50$, an amplitude of 0.5 meters and an initial phase angle of -120°.

c. In <u>space</u> and <u>time</u>, moving along the (+) z-direction with a velocity of 100 cm/s, a wavelength of 1 cm, and an amplitude of 10 cm. Assume the displacement is zero for initial conditions $z = t = 0$.